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Comparison of visual servoing approaches on straight lines

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***Abstract*—In this paper, we compare three approaches of visual servoing on four lines. We present in detail the calculation of the interaction matrix for a 3D line and its projection onto an image. The visual features consists of the orientation of the projected line in the image space as well as its orthogonal distance to the origin of the coordinate system centered in the image space. The three compared approaches are the instantiation of the interaction matrix at the current point, at the reference point and the interaction matrix that is the center of the first two interaction matrices. The results we have obtained show that the use of the latter approach is more adequate to converge towards the reference state.**

I. INTRODUCTION

Visual servoing techniques consist in controlling the movements of a dynamic system from information provided by a vision sensor (cameras). Many studies have been undertaken in this field for several years[1]. In fact, it is constantly boosted by new possibilities in terms of computing power, performance of imagers or new robotic applications. Nevertheless, a number of substantive issues are generic to the discipline and can be dealt with independently of any practical considerations. Two main aspects have a great impact on the behavior of any visual servoing scheme: the selection of the visual features used as input of the control law and the form of the control scheme. As for the visual features, they can be selected in the image space (point coordinates, parameters representing straight lines or ellipses, [5], [9], [3], [6], [1]),in the Cartesian space (pose, coordinates of 3D points [13], [14]), or composed of a mixture of both kinds of features attempting to incorporate the advantages of both image-based and position based methods [13], [7], [2]. As for the choice of the control law [5], [12], [2], it affects the behavior of the selected visual features (local or global exponential decrease, second order minimization, ...) and may lead, or not, to local minima and singularities [3]. In this paper we study and compare three approaches of visual servoing on straight lines. Each method differs by the instanciated interaction matrices: *(1) Computed at the desired state,* (2) computed at the current state and *(3) computed as the mean of the previous ones. Besides, we provide calculation details on the algebraic derivation of the interaction matrix formula. We use the orthogonal distance to the origin and the orientation as visual features.*

A. *Visual servoing on a straight line:*

A 3D straight line can be seen as the intersection of two 3D planes. The projection of this line in the image is also a line. Without loss of generality, one of the two planes that describes the 3D line is chosen to be the plane that contains the camera center and the 2D projected line.

$$H(X, P_0) = \begin{cases} A_1 X + B_1 Y + C_1 Z = 0 \\ A_2 X + B_2 Y + C_2 Z + D_2 = 0 \end{cases} \tag{1}$$

A_1*,* B_1*,* C_1*,* A_2*,* B_2*,* C_2 *and* D_2 *are the parameters of the planes. equation:*

The projected 3D line in the image is given by the following equation:

$$Ax + By + C = 0, \tag{2}$$

where $A = A_1$*,* $B = B_1$ *and* $C = C_1$*.* $\mathbf{x} = (x, y)$ *being the coordinate of the 2D points in the image coordinate frame. We represent the 2D line by the visual features* $\mathbf{p}_i = (\rho, \theta)^T$*. We define* $g(\mathbf{x}, \mathbf{p}_i)$ *the function of the 2D primitive as follows:*

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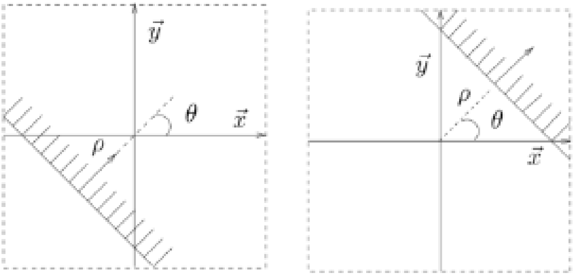

$\rho < 0$ $\rho > 0$

Fig. 1. lines representation(ρ, θ)

$g(\mathbf{x}, \mathbf{p}_i)$ represents the equation of a line with the parameters (ρ, θ)

$$g(\mathbf{x}, \mathbf{p}_i) = xcos\theta + ysin\theta - \rho \tag{3}$$

$$\theta = arctan(B/A)$$

$$\rho = \frac{C}{\sqrt{A^2 + B^2}}$$

$$\dot{g}(\mathbf{x}, \mathbf{p}_i) = 0 \tag{4}$$

$$\frac{\partial g}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{p}_i) + \frac{\partial g}{\partial \mathbf{p}_i}(\mathbf{x}, \mathbf{p}_i) = 0 \tag{5}$$

$$\dot{\rho} + (x \sin\theta - ycos\theta) + \dot{x}\cos\theta + \dot{y}sin\theta = 0 \tag{6}$$

write x with the parameters y, ρ, θ

$$x = f(y, \rho, \theta)$$

$$x = \frac{\rho}{\cos\theta} - y\tan\theta \tag{7}$$

$$A = -x\dot{\theta}sin\theta + y\dot{\theta}cos\theta - \dot{\rho} \tag{8}$$

$$= -(\frac{\rho}{cos\theta} - ytan\theta)\dot{\theta}sin\theta + y\dot{\theta}cos\theta - \dot{\rho} \tag{9}$$

$$= y\dot{\theta}tan\theta sin\theta - tan\theta\rho\dot{\theta} + y\dot{\theta}cos\theta - \dot{\rho} \tag{10}$$

the interaction matrix of a point
in the case of a point of coordinates $x = (x, y)$ in the image space and of depth Z in the reference of the camera, the interaction matrix is given by:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} -\frac{1}{Z} & 0 & \frac{y}{Z} & xy & -(1+x^2) & y \\ 0 & -\frac{1}{Z} & \frac{y}{Z} & (1+y^2) & -xy & -x \end{bmatrix} * v \tag{11}$$

and v it's speed

$$v = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]'$$

$$-\frac{\dot{\theta}}{cos\theta}y + \dot{\rho} + \rho tan\theta\dot{\theta} = yK_1(p_i, p_0)v + k_2(p_i, p_0) * v \tag{12}$$

$$x\dot{\theta}sin\theta + y\dot{\theta}cos\theta - \rho = -(\frac{\rho}{cos\theta} - ytan\theta) \tag{13}$$

$$\begin{pmatrix} K_1 \\ k_2 \end{pmatrix} = \begin{bmatrix} -\lambda_1 cos\theta & \lambda_2 sin\theta & \lambda_1\rho & -\rho & -\rho tan\theta & -\frac{1}{cos\theta} \\ -\lambda_2 cos\theta & \lambda_2 sin\theta & \lambda_2\rho & -sin\theta & \frac{cos\theta + \rho^2}{cos\theta} & \rho tan\theta \end{bmatrix} \tag{14}$$

$$\lambda_1 = Atan\theta + B \tag{15}$$

$$\lambda_2 = \frac{A\rho}{cos\theta} + C \tag{16}$$

$$\tag{17}$$

$$\dot{\theta} = K_1(p1, p0)cos\theta v \tag{18}$$

$$\dot{\rho} = (K_2(p_1, p_{2)} + K_2(p_1, p_0)\rho\sin\theta) * v \tag{19}$$

the interaction matrix $(L_\rho^T L_\theta^T)$

$$\begin{pmatrix} L_\rho^T & L_\theta^T \end{pmatrix} = \begin{bmatrix} -\lambda_\rho cos\theta & -\lambda_\theta cos\theta \\ \lambda_\rho sin\theta & \lambda_\theta sin\theta \\ \lambda_\rho\rho & -\lambda_\theta\rho \\ (1-\rho)sin\theta & -\rho\cos\theta \\ -(1+\rho^2)cos\theta & -\rho\sin\theta \\ 0 & 1 \end{bmatrix} \tag{20}$$

$$\lambda_\rho = -A\cos\theta + B\rho\sin\theta + C \tag{21}$$

$$\lambda_\theta = Bcos\theta - Asin\theta \tag{22}$$

$$L_s = (L_\rho^T, L_\theta^T)^T \tag{23}$$

II. SERVOING CONTROL

We will now show how we can build a control from 2D visual information. To do this, we consider the vector $\mathbf{s}$ *which represents the measurements extracted from the image. For our case these measurements correspond to the vector .* $\mathbf{s} = (\rho, \theta)^T$.

We will minimize the error defined by the following relation:

$$e(t) = \mathbf{s} - \mathbf{s}^* \tag{24}$$

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$\mathbf{s}$ *is the current measure and* $\mathbf{s}^*$ *is the reference measure. We need to determine the relationship between the speed of the visual cues* $\dot{\mathbf{s}}$ *and the velocity of the camera* v*:*

$$\dot{\mathbf{s}} = L_{\mathbf{s}} v \tag{25}$$

Where v *is a translational speed and rotation vector in 3D* $v = (v_x, v_y, v_z, w_x, w_y, w_z)^T$*. En utilisant (**??**) et (**??**), we obtain the relation between the speed of the camera and the variation in time of the error:*

$$\dot{e} = L_{\mathbf{s}} v \tag{26}$$

*using the equation (**??**) we will cancel the error with the introduction of a proportional gain* λ

$$v = -\lambda L_{\mathbf{s}}^{+} e \tag{27}$$

where $L_{\mathbf{s}}^{+}$ *is the pseudo-inverse of* $L_{\mathbf{s}}$*. continuing the development we find*

$$v = -\lambda L_{\mathbf{s}}^{+} (\mathbf{s} - \mathbf{s}^*) \tag{28}$$

*We close the servoing loop using (**??**) and assuming that we realize perfectly* v

$$\dot{e} = -\lambda L_{\mathbf{s}} L_{\mathbf{s}}^{+} e \tag{29}$$

This last equation characterizes the scheme followed. The following relation will make it possible to define the representation of the servoing scheme for four lines .

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\rho}_1 \\ \dot{\theta}_2 \\ \dot{\rho}_2 \\ \dot{\theta}_3 \\ \dot{\rho}_3 \\ \dot{\theta}_4 \\ \dot{\rho}_4 \end{pmatrix} = \begin{pmatrix} L_{\theta_1} \\ L_{\rho_1} \\ L_{\theta_2} \\ L_{\rho_2} \\ L_{\theta_3} \\ L_{\rho_3} \\ L_{\theta_4} \\ L_{\rho_4} \end{pmatrix} v = L\, v \tag{30}$$

In this case the use of four lines is justified by obtaining eight equations for the six unknowns of the velocity of the camera. . The L *matrix can be computed in different ways: (1)* $L_{\mathbf{s}}$ *At the current point* $\mathbf{s}$*, (2)* $L_{\mathbf{s}^*}$ *at the reference point* $\mathbf{s}^*$ *or (3)* $\frac{L_{\mathbf{s}} + L_{\mathbf{s}^*}}{2}$ *as the center between the first two. The following section compares these three different approaches in terms of convergence.*

III. EXPERIMENTAL RESULTS

The visual servoing approach is tested on lines using a free camera (6 degrees of freedom) and four lines. We compare three approaches of visual servoing on lines: (1) An approach that uses the L interaction matrix taken in the current state. *(2) An approach that uses the interaction matrix* L^* *taken at the reference state.* (3) An approach that uses the interaction matrix $\frac{L+L^*}{2}$ which is the center of the previous interaction matrices. In the reference state, the camera is aligned with the world marker and at the initialization state the camera is rotated by 5 degrees around its optical axis Z. The focal length of the camera is $f = 1500^1$.We take lines belonging to the plane. $X+Y+Z = 20$. They go through the following points: Line 1 $([3,3,14];[4,6,10])$, Line 2 $([4,6,10];[-2,2,20])$, Line 3 $([-2,2,20];[-2,-2,24])$, Line 4 $([-2,-2,24];[3,3,14])$. For the servoing we take $\lambda = 0.001$ and we put a condition of stop of the servoing loop whenever the norm of the differences of visual features is lower than 0.01. The results are shown in the figures **??**, **??**, **??**. Figure **??** represents the image of the four lines in the initial state with respect to their image in the reference state. From the experiments conducted, it appears that the strategy that uses $\frac{L+L^*}{2}$ as the interaction matrix presents the best results in terms of accuracy and achievement of the state of reference. We observe that the speed of convergence is slow (after 2000 iterations). A higher $lambda$ does not allow convergence. We have also noticed that initial states too far from the reference state (more than 10 degrees in rotation and more than 300 mm in translation) do not allow convergence to the initial state. We also run experiments on the variation of the eigenvalues of $() - \lambda L_{\mathbf{s}} L_{\mathbf{s}}^{+})$ and observe that we have the whole eight eigen values with negative real parts. Six of them are equal to -10^{-3}. The two other eigenvalues are of order -10^{-20}. With such small absolute real part, the visual feature error decreases slowly. In future work, it would be interesting to study these cases of divergence and to develop a methodology to accelerate convergence even for larger movements.

IV. CONCLUSION

Comparing different four-line visual servoing approaches, we have found that the approach where we use the interaction matrix $fracL + L^*2$ which is the center of the interaction matrices at current and reference point provides better results in terms

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[1] The units of distances are millimeters and the angles are degrees.

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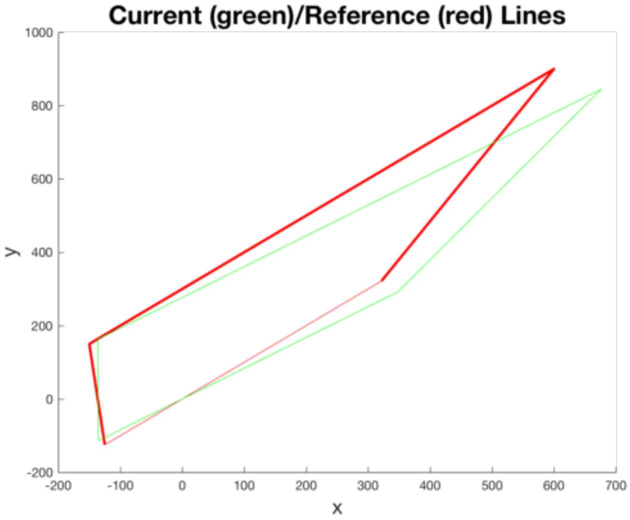

Fig. 2. Initial state. In red the lines references and in green the lines seen in the initial state.

of accuracy and speed of convergence. The use of interaction matrix at one of the two points mentioned above is not recommended in this case. The study of eigenvalues allow us to explain why the convergence rate is slow. In future works, we will study the different cases of divergences as well as the variations of the singular values of matrices LL^+ which determine the stability of the servo process.

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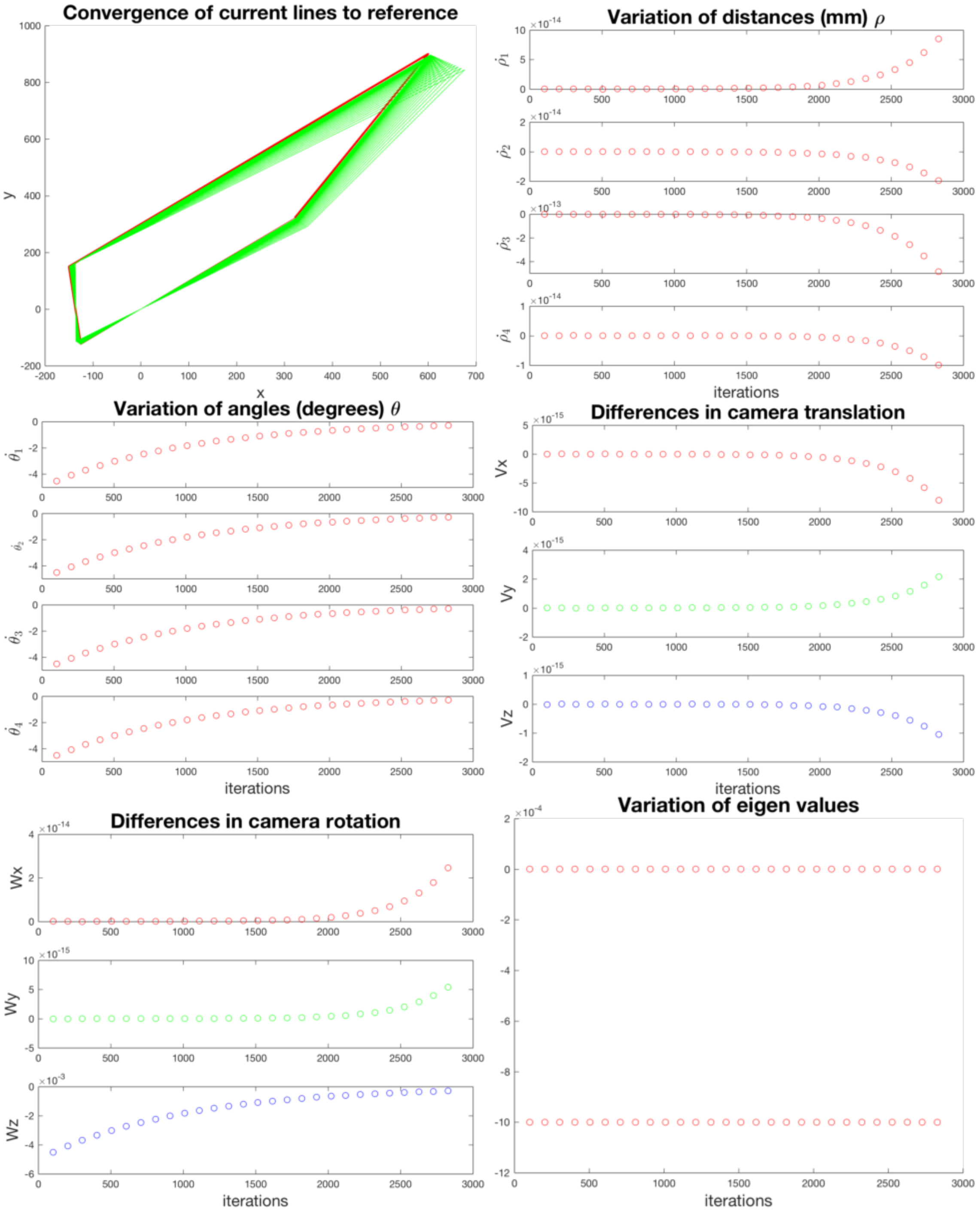


Convergence of current lines to reference

Variation of distances (mm) ρ

Variation of angles (degrees) θ

Differences in camera translation

Differences in camera rotation

Variation of eigen values

Fig. 3. Results of the experiment with L.

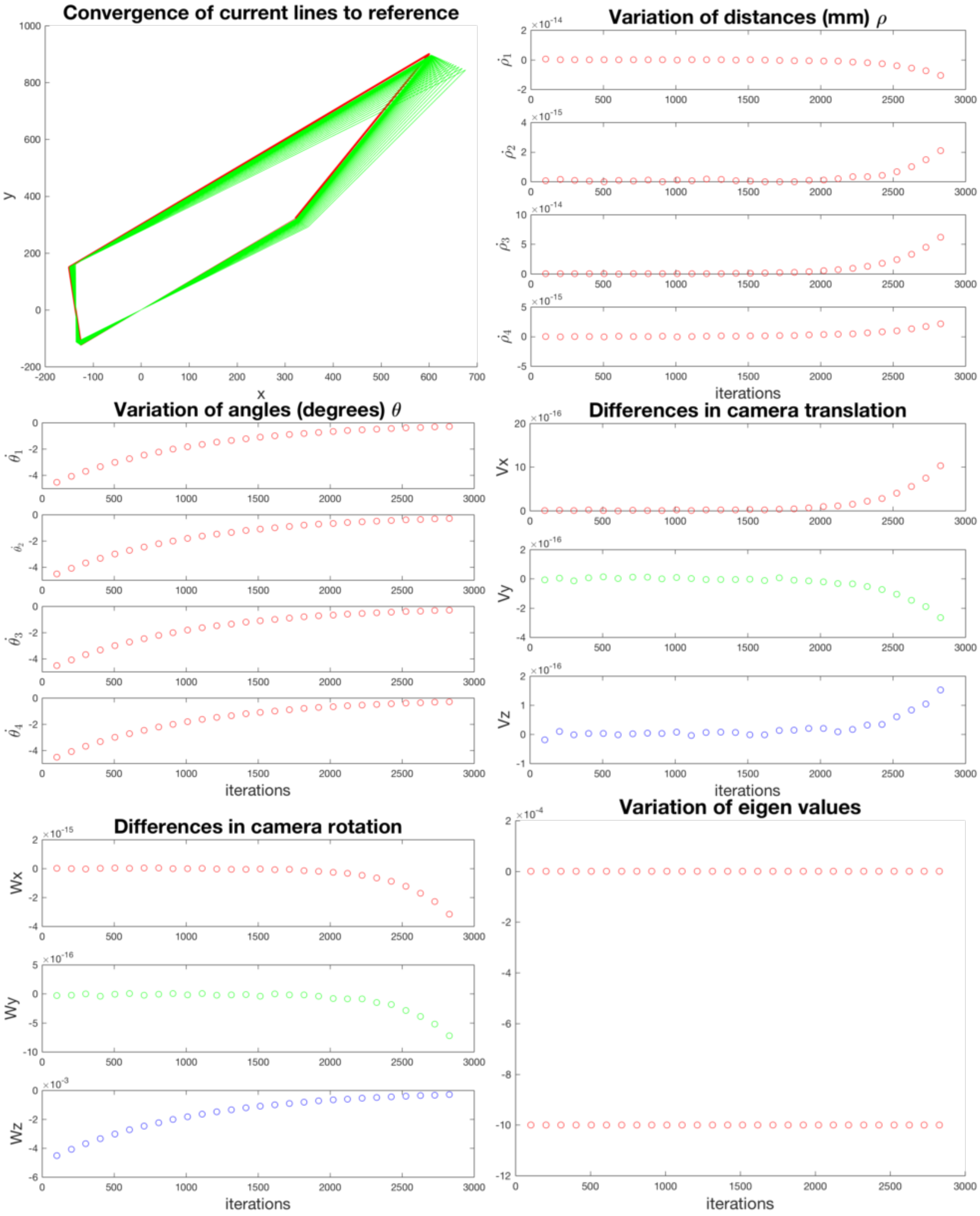
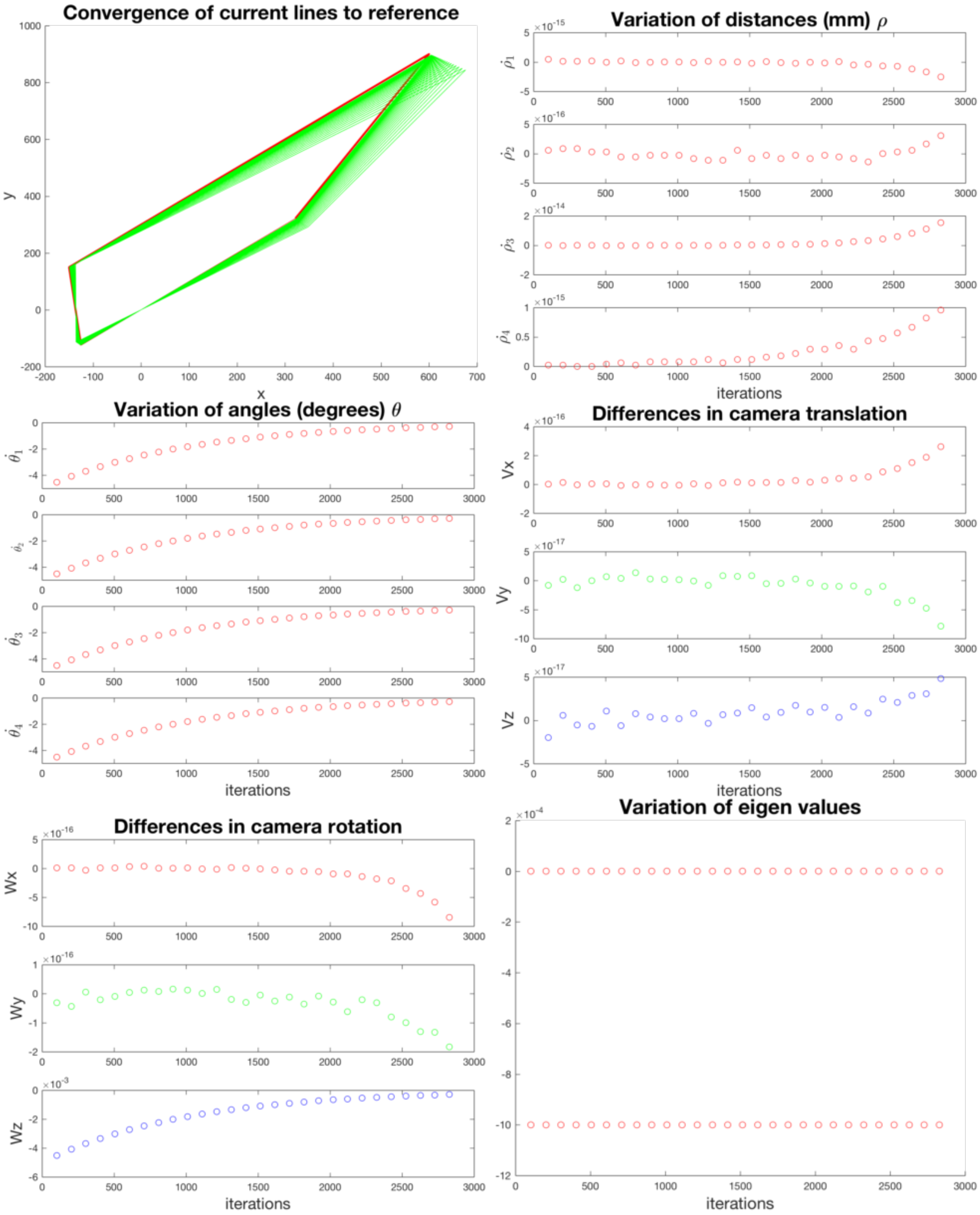


Fig. 4. Results of the experiment with L^*.


Convergence of current lines to reference

Variation of distances (mm) ρ

Variation of angles (degrees) θ

Differences in camera translation

Differences in camera rotation

Variation of eigen values

Fig. 5. Results of the experiment with $\frac{L+L^*}{2}$.