Conférence Internationale en Automatique & Traitement de Signal (ATS-2018)
Proceedings of Engineering and Technology – PET
Vol.36 pp.17-21

New methodology for thermal regulation of CHP systems

Mustapha Ait Rami*

***Abstract*— This paper deals with thermal regulation of a CHP system (Combined Heat and Power also known as Cogeneration system). For this kind of system we present a dynamic model that describes the heat transfer between its main components. We show that such model inherits the positivity feature from the underlying thermal system. The regulation purpose is to track a desired temperature for the hot water in the secondary water circuit. In order to improve the decay rate of the thermal system, a stability analysis is provided by taking into account the system's flow rates. Based on positivity consideration, we show that the proposed regulation problem can be solved via an adequate Linear Programming (LP) formulation.**

Keywords: CHP system, positive systems, thermal systems, linear programming.

I. INTRODUCTION

Regulation of fluid outlet temperature at desired values is of great importance in many domestic and industrial applications. This can be achieved by transporting a fluid through ducts to heat exchangers with adequate flow rates. Modeling and control of thermal systems is a topic of great practical importance. Many different methods to control temperature and heat in a building have been proposed [14], [2], [3], [6], [7], [15]. Our treatment for a thermal CHP system is mainly based on the theory of positive systems (whose states are nonnegative whenever its initial condition are nonnegative). For a wide overview on positive systems, see for instance, [5], [9], [8].

This paper treats the thermal regulation of a Combined Heat and Power (CHP) system also known as Cogeneration system. For such system a dynamic model is proposed which describes the heat transfer between the main components: the Stirling engine, the primary water circuit and the secondary water circuit. Such model inherits the positivity of the underlying thermal system. The aim is to track a desired temperature for the hot water in the secondary water circuit. For this purpose, a positivity-based methodology is provided in order to improve the decay rate of the system. First, a stability analysis is performed by taking into account some crucial components of the system, that is, its tunable parameters represented by the flow rates. Such important parameters of a CHP system may not all be controlled, but rather can be a priori tuned in order to achieve a good tracking performance. We show that this tuning problem can be solved via an adequate LP formulation. Furthermore, based on positivity consideration we show how the fuel flow of the Stirling engine can be selected in order to achieve a desired temperature reference.

* The author is with ENSIT Tangier, Morocco. email: musta.ait.rami@gmail.com

This paper is organized as follows: Section II presents a state-space thermal model of a CHP system. In section III, we propose a positivity framework for CHP system's stability. In section IV, we treat the stability performance and the thermal regulation by taking into account the flow rates as tunable parameters. Finally, a characterization of the full physical thermal model is provided in Appendix.

Notations $\mathbb{R}^{n\times m}$ denotes the set of real matrices of size $n\times m$ and $\mathbb{R}_+^{n\times m}$ denotes the set of real matrices with nonnegative entries. For a real matrix or a vector, $M>0$ means that its components are positive, and $M\geq 0$ means that its components are nonnegative. M^T denotes the transpose of M.

II. THERMAL CHP SYSTEM AND PROBLEM FORMULATION

We consider a state-space thermal model which characterizes a CHP system as depicted in Figure 1.

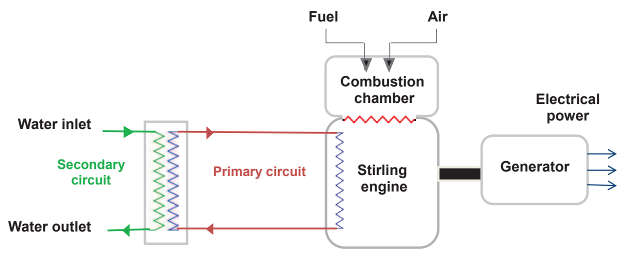

Fig. 1. Structure of a CHP unit with a Stirling engine

The thermal model of a CHP system is elaborated by taking into account the overall steady-state energy balance: energy balance in the engine, energy balance at the main exchanger for cooling water circuit and energy transport equation for cold water and hot water (more details on the physical model are given in the Apprendix). The state variables are: the average engine temperature T_{eng}, the outlet cold water temperature $T_{cw,o}$, the temperature of the inlet cold water $T_{cw,i}$ and the outlet temperature of the hot water $T_{hw,o}$. The system's manipulated input is the fuel flow rate f_{eng} entering the combustion chamber in the Stirling engine. The structure of the overall state-space thermal model is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + d(t) \\ y(t) = Cx(t) \end{cases} \qquad (1)$$

where $x\in\mathbb{R}^4$ is the state and $y\in\mathbb{R}$ is the output measurements such that $x = [T_{eng}\ T_{cw,o}\ T_{cw,i}\ T_{hw,o}]^T$, $y = T_{hw,o}$. The control input u represents the fuel flow rate f_{eng} in the engine and $d\in\mathbb{R}^4$ represents a mesurable exogenous signal. The

ISSN 2356-5608

matrices $A \in \mathbb{R}^{4\times4}$, $B \in \mathbb{R}^4$ $C \in \mathbb{R}^{1\times4}$ and $d \in \mathbb{R}^4$ have the following structure

$$A = \begin{bmatrix} -\alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_3 & -\alpha_4 f_{cw} - \alpha_5 & \alpha_4 f_{cw} & 0 \\ 0 & \alpha_4 f_{cw} & -\alpha_4 f_{cw} - \alpha_5 & 0 \\ 0 & \alpha_6 & 0 & -\alpha_7 f_{hw} - \alpha_6 \end{bmatrix}, \tag{2}$$

$$B^T = \begin{bmatrix} \beta & 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix},$$

$$d^T = \begin{bmatrix} \gamma_1 & 0 & \gamma_2 & \gamma_3 \end{bmatrix},$$

where all the involved physical and thermal parameters are positive $\alpha_i > 0$, $f_{cw} > 0$, $f_{hw} > 0$, $\beta > 0$ and $\gamma_i > 0$. The parameters f_{cw}, f_{hw} represent respectively the flow rates of the cold and hot water. The control input $u = f_{eng}$ is nonnegative where $f_{eng} > 0$ is the fuel flow rate.
For the thermal regulation purpose, the flow rates parameters of the thermal model (1) have to be tuned. Thereby, allowing considerable freedom in the assignment of a decay rate of the system. Indeed, this primordial concern for assigning reasonable values to some manipulated parameters of the thermal model can increase tracking performance. Henceforth, our attempt is to provide satisfactory answers to the following questions:

- How can one tune the flow rates f_{cw}, f_{hw} in order to increase the decay rate of system (1)?
- How can one derive a nonnegative control (fuel flow rate) to track a desired temperature for the output $y = Cx = T_{hw,o}$ depending on the tuned flow rates f_{cw}, f_{hw}?

Keeping in mind that the underlying thermal model (1) is positive by nature, our forthcoming treatment will be performed within a positivity-framework. To this end, some key role tools and preliminary results will be introduced.

III. Positivity and Stability

First, fundamental stability and positivity properties related to positive systems are presented.

Consider the following linear system

$$\dot{x}(t) = Mx(t), x(0) = x_0 \in \mathbb{R}^n_+, \tag{3}$$

where $M \in \mathbb{R}^{n\times n}$ is a constant matrix.
Definition 3.1: System (3) is said to be positive if its trajectories are nonnegative for all nonnegative initial conditions.
Definition 3.2: A real matrix M is called a Metzler matrix if its off-diagonal entries are nonnegative ($m_{ij} \geq 0,\ i \neq j$).
Note that the dynamic matrix A of the thermal system (1) is Metzler. Then, the internal positivity of such system (with $u = 0$, $d = 0$) can be easily checked based on the following result (see [11]).
Lemma 3.3: System (3) is positive if and only if M is a Metzler matrix.
In the sequel, we shall use the following stability result which has been established in different contexts, see for instance [11], [13], [1].
Lemma 3.4: For any Metzler matrix M, we have that $e^{tM} \geq 0$ for all $t \geq 0$. Moreover, the following statements are equivalent

(i) M is Hurwitz.
(ii) There exists a positive vector $\lambda > 0$ such that $M\lambda < 0$.

The following comparison result is needed for our next result.
Lemma 3.5: Consider the following system

$$\dot{z}(t) = Mz(t) + f(t),\ z(0) = z_0 \in \mathbb{R}^n_+, \tag{4}$$

where M is a Metzler matrix. If $f(t) \geq 0$ for all $t \geq 0$, then the states of system (4) is nonnegative $z(t) \geq 0$ for all $t \geq 0$. Moreover, $z(t)$ is bounded from below $z(t) \geq \tilde{z}(t)$ for all $t \geq 0$, where $\tilde{z}(t)$ is the solution to the differential equation

$$\dot{\tilde{z}} = M\tilde{z},\ \tilde{z}(0) = z_0.$$

Proof: This can be shown by using the fact that e^{tM} is nonnegative for all $t \geq 0$ (see Lemma 3.4) and by using the following expression of the solution to the equation (4)

$$z(t) = e^{tM}z(0) + \int_0^t e^{(t-s)M} f(s)ds.$$

■

Based on the previous result some drawbacks related to improving stability performance for system (1) through the use of nonnegative control are first revealed.
Proposition 3.6: Any kind of nonnegative control cannot increase the decay rate of the thermal system (1).
Proof: First recall that the dynamic matrix A of the thermal system (1) is Metzler. Then, the argument line can be deduced from the comparison result of Lemma 3.5. Effectively, let $x_u(t)$ be the trajectory of system (1) under a nonnegative control $u(t) \geq 0$ and $x_0(t)$ be the trajectory with zero input $u = 0$. Since $B \geq 0$ then by Lemma 3.5 we have that $x_u(t) \geq x_0(t)$ for all $t \geq 0$. Consequently, the decay rate of $x_u(t)$ is the same or less than the decay rate of $x_0(t)$. ■

IV. Enhanced Performance

In this part, stability with a prescribed decay rate for system (1) is investigated under tuned flow rates.
Recall that we have shown previously that any kind of nonnegative control cannot increase the decay rate of the thermal system. As a remedy, the flow rates f_{cw}, f_{hw} can be tuned in order to improve the stability performance.

In the sequel, the following Lemma will play a technical key role.
Lemma 4.1: For any given matrix M, the following statements are equivalent

(i) There exists a positive vector $\gamma > 0$ such that $M\gamma < 0$.
(ii) There exists a positive vector $\lambda = [\lambda_1, \ldots, \lambda_n]^T > 0$ such that $M\lambda < 0$ and $\lambda_i \neq \lambda_j$ for any $i \neq j$.

Proof: Since we have strict inequality $M\gamma < 0$, the equivalence can be shown by using a small perturbation of the components of the vector $\gamma > 0$ in the statement (i). ■
Now, we are in a place to state the following result.
Theorem 4.2: Let $A \in \mathbb{R}^{4\times4}$ be the dynamic matrix of the thermal system (1) defined by (2). Then there exist bounded positive flow rates $\underline{f_{cw}} \leq f_{cw} \leq \overline{f_{cw}}$, $\underline{f_{hw}} \leq f_{hw} \leq \overline{f_{hw}}$ (where

ISSN 2356-5608

Conférence Internationale en Automatique & Traitement de Signal (ATS-2018)
Proceedings of Engineering and Technology – PET
Vol.36 pp.17-21

$\underline{f_{cw}}$,$\overline{f_{cw}}$, $\underline{f_{hw}}$ and $\overline{f_{hw}}$ are prescribed bounds) such that the real part of the eigenvalues of A are less that $-r<0$ (where r is a prescribed decay rate); if and only if either one of the following LP problems ((5) or (6)) is feasible in the scalar variables λ_1, λ_2, λ_3, λ_4 and y_1, y_2

$$\begin{cases} \begin{bmatrix} r-\alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \alpha_3 & r-\alpha_5 & 0 & 0 & \alpha_4 & 0 \\ 0 & 0 & r-\alpha_5 & 0 & \alpha_4 & 0 \\ 0 & -\alpha_6 & 0 & r-\alpha_6 & 0 & -\alpha_7 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ y_1 \\ y_2 \end{bmatrix} < 0, \\ \lambda_1>0,\ \lambda_2>0,\ \lambda_3>0,\ \lambda_4>0, \\ \lambda_3-\lambda_2>0, \\ y_1>0,\ y_2>0, \\ \underline{f_{cw}}(\lambda_3-\lambda_2) \le y_1 \le \overline{f_{cw}}(\lambda_3-\lambda_2),\ \underline{f_{hw}}\lambda_4 \le y_2 \le \overline{f_{hw}}\lambda_4. \end{cases} \tag{5}$$

$$\begin{cases} \begin{bmatrix} r-\alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \alpha_3 & r-\alpha_5 & 0 & 0 & \alpha_4 & 0 \\ 0 & 0 & r-\alpha_5 & 0 & \alpha_4 & 0 \\ 0 & -\alpha_6 & 0 & r-\alpha_6 & 0 & -\alpha_7 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ y_1 \\ y_2 \end{bmatrix} < 0, \\ \lambda_1>0,\ \lambda_2>0,\ \lambda_3>0,\ \lambda_4>0, \\ \lambda_3-\lambda_2<0, \\ \underline{y_1}<0,\ y_2>0, \\ \overline{f_{cw}}(\lambda_3-\lambda_2) \le y_1 \le \underline{f_{cw}}(\lambda_3-\lambda_2),\ \underline{f_{hw}}\lambda_4 \le y_2 \le \overline{f_{hw}}\lambda_4. \end{cases} \tag{6}$$

Moreover, the desired flow rate values can be recovered from any feasible LP problem (5) or (6) as

$$f_{cw} = (\lambda_3-\lambda_2)^{-1}y_1 \text{ and } f_{hw} = \lambda_4^{-1}y_2 \tag{7}$$

Proof: The necessity part can be proved as follows. Assume that there exist bounded positive flow rates $\underline{f_{cw}} \le f_{cw} \le \overline{f_{cw}}$, $\underline{f_{hw}} \le f_{hw} \le \overline{f_{hw}}$ such that the real part of the eigenvalues of the matrix A defined by (2) are less than $-r<0$, then by Lemma 3.4 and Lemma 4.1, this is equivalent to the existence of a positive vector $\lambda=[\lambda_1,\ldots,\lambda_n]^T>0$ such that $(A+rI)\lambda<0$ with $\lambda_2 \neq \lambda_3$. Thus, two cases can happen $\lambda_2>\lambda_3$ or $\lambda_2<\lambda_3$.

If $\lambda_2>\lambda_3$, then by tacking the change of variable $y_1 = f_{cw}(\lambda_3-\lambda_2)^{-1}$ and $y_2 = f_{hw}\lambda_4^{-1}$ we get by some manipulations and rearrangement the LP problem (5) Also, the case $\lambda_2<\lambda_3$ with this change of variables $y_1 = f_{cw}(\lambda_3-\lambda_2)^{-1}$ and $y_2 = f_{hw}\lambda_4^{-1}$ leads to the LP problem (6).

The sufficiency part follows the same line of argument. ■

Next, the regulation problem under consideration, is to determine a nonnegative control signal $u \ge 0$ such that the outlet hot water temperature $y = Cx = T_{hw,o}$ tracks a desired constant reference y_{ref}.

Assume that the exogenous signal $d(t)$ is measurable and goes to a steady value d^* (which is the case for the termal system). Then, a simple regulation strategy can be based on the following feed-forward control which will be generated based upon adequate hot and cold water flows.

$$u(t) = -\frac{y_{ref}+CA^{-1}d(t)}{CA^{-1}B}. \tag{8}$$

If the matrix A is stable, then the system goes to a steady-state x^*, u^* at the equilibrium:

$$Ax^*+Bu^*+d^*=0, \tag{9}$$

As $u^* = -\frac{y_{ref}+CA^{-1}d^*}{CA^{-1}B}$, by multiplying (9) by the matrix CA^{-1} we can deduce easily that $Cx^* = y_{ref}$.

The question is how to ensure that the tracking control law (8) is nonnegative. This nonnegativity condition on the control is a real physical constraint since the fuel flow rate must be a nonnegative physical quantity. Note that in general the design of a negative is a hard problem.

In the sequel, we shall use the following general result with connection to Metzler matrices that are Hurwitz (see for instance [4]).

Lemma 4.3: Let M be a Metzler matrix, then the following statements are equivalent

(i) M is Hurwitz.
(ii) $M^{-1} \le 0$.

Now, the following result provides linear constraints conditions for the nonnegativity of the feed-forward control (8) explicitly in terms of the data matrices A,B and C. Later, such result can be combined with the previous stability performance result of Theorem 4.2 in order to increase the decay rate of the system.

Theorem 4.4: Let $A \in \mathbb{R}^{n\times n}$ be a Metzler and Hurwitz matrix, $B \in \mathbb{R}^n$, $C \in \mathbb{R}^{1\times n}$ such that $B \ge 0$, $C \ge 0$ and $CA^{-1}B \neq 0$. Assume that $d(t) \in \mathbb{R}^n$ is a bounded vector $\underline{d} \le d(t) \le \overline{d}$ for all $t \ge 0$ with given constant vectors $\underline{d} \ge 0, \overline{d} \ge 0$. Then, the control signal $u(t) = -\frac{y_{ref}+CA^{-1}d(t)}{CA^{-1}B}$ is positive and bounded $0 \le u(t) \le \overline{u}$ for all $t \ge 0$ with prescribed constant upper bound $\overline{u}>0$, if there exists a vector $\lambda \in \mathbb{R}^n$ satisfying the following linear constraints

$$\begin{cases} A\lambda \le -\overline{d} \\ A\lambda \ge -\overline{u}B - \underline{d} \\ C\lambda = y_{ref} \end{cases} \tag{10}$$

Proof: First, note that the statement: A is a Metzler and Hurwitz matrix implies by Lemma 4.3 that $A^{-1} \le 0$. This implies $CA^{-1}B<0$ since $B \ge 0$, $C \ge 0$. Thus, in order to see that $u \ge 0$ it suffices to show that $y_{ref}+CA^{-1}d(t) \ge 0$ for all $t \ge 0$. Hence, by using the condition $A\lambda \le -\overline{d}$ which by simple manipulation yields $-C\lambda \le CA^{-1}\overline{d}$ which by using the fact that $C\lambda = y_{ref}$, implies $y_{ref} \le CA^{-1}\overline{d}$. Consequently, by tacking into account that $d(t) \le \overline{d}$ which leads to $CA^{-1}\overline{d} \le CA^{-1}d(t)$ (multiply by $CA^{-1} \le 0$). Hence, we can see that $y_{ref}+CA^{-1}d(t) \ge y_{ref}+CA^{-1}\overline{d} \ge 0$ for all $t \ge 0$.

Next, let us prove that $u \le \overline{u}$ which is equivalent to $-y_{ref} \ge CA^{-1}(\overline{u}B+d(t))$ (due to the fact that $CA^{-1}B<0$). Since $\underline{d} \le d(t)$ it holds $CA^{-1}\underline{d} \ge CA^{-1}d(t)$ (multiply by $CA^{-1} \le 0$) and so that we only need to show that the inequality $-y_{ref} \ge CA^{-1}(\overline{u}B+\underline{d})$ holds. Let λ satisfies the condition $A\lambda \ge -\overline{u}B-\underline{d}$ which by using the fact that $A^{-1} \le 0$ implies $-C\lambda \ge CA^{-1}(\overline{u}B+\underline{d})$. By tacking into account the other condition $C\lambda = y_{ref}$ we obtain $-y_{ref} \ge CA^{-1}(\overline{u}B+\underline{d}) \ge CA^{-1}(\overline{u}B+d(t)$ and the proof is complete. ■

ISSN 2356-5608

Next, we shall show how the previous result can be applied to the thermal system (1). Note that in order to preserve stability with a given decay rate and recover the associated flow rates, the linear inequality condition $A\lambda \leq -\overline{d}$ is replaced with $(A+rI)\lambda \leq -\overline{d}$ which implies by Lemma 3.4 that $A+rI$ is a Hurwitz matrix.

Now, the following result presents LP conditions for the regulation problem. Such conditions are numerically reliable and easily checkable by using linear programming.

Corollary 4.5: Consider the thermal system (1) with bounded exogenous signal $\underline{d} \leq d(t) \leq \overline{d}$ for all $t \geq 0$ with given constant vectors $\underline{d}, \overline{d} \in \mathbb{R}^n_+$. If either one of the following LP problems ((11) or (12)) is feasible in the scalar variables λ_1, λ_2, λ_3, λ_4 and y_1, y_2

$$\begin{cases} \mathscr{A}\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & y_1 & y_2\end{bmatrix}^T \leq -\overline{d}, \\ \mathscr{A}\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & y_1 & y_2\end{bmatrix}^T \geq -\bar{u}B-\underline{d}, \\ C\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}^T = y_{ref}, \\ \lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_3 > 0,\ \lambda_4 > 0, \\ \lambda_3 - \lambda_2 > 0, \\ y_1 > 0,\ y_2 > 0, \\ \underline{f_{cw}}(\lambda_3-\lambda_2) \leq y_1 \leq \overline{f_{cw}}(\lambda_3-\lambda_2),\ \underline{f_{hw}}\lambda_4 \leq y_2 \leq \overline{f_{hw}}\lambda_4. \end{cases} \tag{11}$$

$$\begin{cases} \mathscr{A}\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & y_1 & y_2\end{bmatrix}^T \leq -\overline{d}, \\ \mathscr{A}\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & y_1 & y_2\end{bmatrix}^T \geq -\bar{u}B-\underline{d}, \\ C\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}^T = y_{ref}, \\ \lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_3 > 0,\ \lambda_4 > 0, \\ \lambda_3 - \lambda_2 < 0, \\ y_1 < 0,\ y_2 > 0, \\ \overline{f_{cw}}(\lambda_3-\lambda_2) \leq y_1 \leq \underline{f_{cw}}(\lambda_3-\lambda_2),\ \underline{f_{hw}}\lambda_4 \leq y_2 \leq \overline{f_{hw}}\lambda_4. \end{cases} \tag{12}$$

where the matrix $\mathscr{A}$ is given by

$$\mathscr{A} := \begin{bmatrix} r-\alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \alpha_3 & r-\alpha_5 & 0 & 0 & \alpha_4 & 0 \\ 0 & 0 & r-\alpha_5 & 0 & \alpha_4 & 0 \\ 0 & -\alpha_6 & 0 & r-\alpha_6 & 0 & -\alpha_7 \end{bmatrix},$$

$\underline{f_{cw}}, \overline{f_{cw}}$, $\underline{f_{hw}}$, $\overline{f_{hw}}$ and $\bar{u} > 0$ are prescribed positive bounds and $r > 0$ is a desired decay rate.

Then, the real part of the eigenvalues of the matrix A (defined by (2)) are less that $-r < 0$ with the flow rates recovered from any feasible LP problem (11) or (12) by

$$f_{cw} = (\lambda_3-\lambda_2)^{-1}y_1 \text{ and } f_{hw} = \lambda_4^{-1}y_2; \tag{13}$$

and such that $\underline{f_{cw}} \leq f_{cw} \leq \overline{f_{cw}}$, $\underline{f_{hw}} \leq f_{hw} \leq \overline{f_{hw}}$. Moreover, if $CA^{-1}B \neq 0$, then the control feed-forward signal $u(t) = -\frac{y_{ref}+CA^{-1}d(t)}{CA^{-1}B}$ is positive and satisfies $0 \leq u(t) \leq \bar{u}$ for all $t \geq 0$.

Proof: The proof is straightforward from Theorem 4.4 and follows the same line of argument as for Theorem 4.2. ■

Remark 4.6: It is easy to achieve an optimum fuel consumption by minimizing the upper bound $\overline{u}$ over the linear constraints (11) or (12).

V. Conclusion

In this paper, we have introduced a methodology for the regulation of a CHP thermal system based on positivity framework. The proposed model is parameterized by some tunable parameters: the flow rates which can be fixed adequately in order to increase the decay rate of the thermal system. Furthermore, we have shown how a nonnegative feed-forward control can be designed in order to achieve a desired temperature for the outlet hot water temperature.

Appendix: Physical CHP Termal Model

This section presents a state-space model which describes a thermal CHP system based on its physical and thermal characteristics.

A CHP system contains an external heat source (boiler), a Stirling engine, a generator and a heat exchanger in order to recover the produced heat. Its basic architecture is shown in Figure 1. The heat source is typically a boiler which can be powered by various types of fuel.

By following [10], the engine's steady-state thermal performance is related to the rate of steady-state heat production P_q that is characterized by

$$P_q = \eta_q LHV_{fuel}\dot{m}_{fuel} \tag{14}$$

where η_q is the engine's steady-state thermal efficiency. The quantity $\dot{m}_{fuel}$ is the fuel flow rate and LHV_{fuel} is the fuel lower heating value.

As stated in [10], the thermal energy stored within the engine is characterized by using an aggregate thermal capacitance and an equivalent average engine temperature. The thermal energy balance for the engine and the main exchanger (cooling water circuit) is given by the following equations

$$[MC]_{eng}\frac{dT_{eng}}{dt} = P_q - \dot{Q}_{hx} - \dot{Q}_{loss} \tag{15}$$

$$[MC]_{cw}\frac{dT_{cw,o}}{dt} = [\dot{m}c_p]_{cw}(T_{cw,i}-T_{cw,o}) + \dot{Q}_{hx} \tag{16}$$

where T_{eng} is the engine temperature, $T_{cw,i}$ is the inlet cooling water temperature, $T_{cw,o}$ is the outlet exit temperature of the encapsulated cooling water. $[MC]_{eng}$ is the engine's thermal capacitance, $[MC]_{cw}$ is the thermal capacitance of the encapsulated cooling water and heat exchanger shell, $[\dot{m}c_p]$ is the thermal capacity flow rate associated with the cooling water. The other quantities $\dot{Q}_{hx}$ and $\dot{Q}_{loss}$ represent, respectively, the rate of heat transfer to the cooling water and the rate of heat loss from the system to the surroundings. These quantities are proportional to the temperatures difference as follows

$$\begin{cases} \dot{Q}_{hx} = Ua_{hx}(T_{eng}-T_{cw,o}) \\ \dot{Q}_{loss} = Ua_p(T_{eng}-T_s) \end{cases} \tag{17}$$

where T_s is the surroundings temperature, Ua_{hx} is the overall thermal conductance between the engine and cooling water, and Ua_p is the effective thermal conductance between the engine and the surroundings.

ISSN 2356-5608

Using equations (17), the engine and cooling water energy balance equations ((15) and (16)) can be reexpressed as

$$[MC]_{eng}\frac{dT_{eng}}{dt} = P_q - Ua_{hx}(T_{eng}-T_{cw,o}) - Ua_p(T_{eng}-T_s) \tag{18}$$

$$[MC]_{cw}\frac{dT_{cw,o}}{dt} = [\dot{m}c_p]_{cw}(T_{cw,i}-T_{cw,o}) + Ua_p(T_{eng}-T_s). \tag{19}$$

In the secondary circuit, the energy transport equations for cold and hot water are

$$[MC]_{cw}\frac{dT_{cw,i}}{dt} = [\dot{m}c_p]_{cw}(T_{cw,o}-T_{cw,i}) + Ua_{hxs}(T_{hw,i}-T_{cw,i}) \tag{20}$$

$$[MC]_{hw}\frac{dT_{hw,o}}{dt} = [\dot{m}c_p]_{hw}(T_{hw,i}-T_{hw,o}) + Ua_{hxs}(T_{cw,o}-T_{hw,o}) \tag{21}$$

where $T_{hw,i}$ is the temperature of the inlet hot water in secondary circuit, $T_{hw,o}$ is the outlet temperature of the encapsulated cooling water, $T_{cw,i}$ is the inlet temperature of the cooling water, UA_{hxs} is the overall thermal conductance between the cooling water and heating water, $[MC]_{hw}$ is the thermal capacitance of the encapsulated heat water and heat exchanger shell and $[\dot{m}c_p]$ is the thermal capacity flow rate of the cooling water.

By gathering all the previous thermal balance equations one can obtain the following CHP thermal model

$$\begin{array}{lcl}
\dot{T}_{eng} & = & \frac{1}{[MC]_{eng}}(\dot{m}_{fuel}LHV - Ua_{hx}(T_{eng}-T_{cw,o}) - Ua_p(T_{eng}-T_s)) \\
\dot{T}_{cw,o} & = & \frac{1}{[MC]_{cw}}([\dot{m}c_p]_{cw}(T_{cw,i}-T_{cw,o}) + Ua_p(T_{eng}-T_s)) \\
\dot{T}_{cw,i} & = & \frac{1}{[MC]_{cw}}([\dot{m}c_p]_{cw}(T_{cw,o}-T_{cw,i}) + Ua_{hxs}(T_{hw,i}-T_{cw,i})) \\
\dot{T}_{hw,o} & = & \frac{1}{[MC]_{hw}}([\dot{m}c_p]_{hw}(T_{hw,i}-T_{hw,o}) + Ua_{hxs}(T_{cw,o}-T_{hw,o}))
\end{array} \tag{22}$$

By defining $x = [T_{eng} \quad T_{cw,o} \quad T_{cw,i} \quad T_{hw,o}]^T$ as state variable, $u = \dot{m}_{fuel}$ as control input and $y = T_{hw,o}$ as output, then a state-space thermal model

$$\begin{cases} \dot{x} = Ax + Bu + d \\ y = Cx \end{cases} \tag{23}$$

can be characterized by

$$A = \begin{bmatrix} \frac{-Ua_{hxs}-Ua_p}{[MC]_{eng}} & 0 & \frac{Ua_{hxs}}{[MC]_{eng}} & 0 \\ \frac{Ua_{hxs}}{[MC]_{cw}} & \frac{-[\dot{m}c_p]_{cw}-Ua_{hxs}}{[MC]_{cw}} & \frac{[\dot{m}c_p]_{cw}}{[MC]_{cw}} & 0 \\ 0 & \frac{[\dot{m}c_p]_{cw}}{[MC]_{cw}} & \frac{-[\dot{m}c_p]_{cw}-Ua_{hxs}}{[MC]_{cw}} & 0 \\ 0 & \frac{Ua_{hxs}}{[MC]_{hw}} & 0 & \frac{-[\dot{m}c_p]_{hw}-Ua_{hxs}}{[MC]_{hw}} \end{bmatrix} \tag{24}$$

$$B = \begin{bmatrix} \frac{\eta_q LHV_{fuel}}{[MC]_{eng}} & 0 & 0 & 0 \end{bmatrix}^T \tag{25}$$

$$d = \begin{bmatrix} \frac{Ua_pT_s}{[MC]_{eng}} & 0 & \frac{Ua_{hxs}T_{hw,i}}{[MC]_{cw}} & \frac{[\dot{m}c_p]T_{hw,i}}{[MC]_{hw}} \end{bmatrix}^T \tag{26}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

REFERENCES

[1] M. Ait Rami and F. Tadeo. Controller synthesis for positive linear systems with bounded controls. *IEEE Transactions on Circuits and Systems*, 54, pp. 151 – 155, 2007.

[2] M. Badami, A. Casetti, P. Campanile, and F. Anzioso. Performance of an innovative 120 kWe natural gas cogeneration system, Applied Energy, vol. 32, 1964, pp. 823833.

[3] L. Giaccone and A. Canova. Economical comparison of CHP systems for industrial user with large steam demand. Applied Energy, vol. 86, 2009, pp. 904914.

[4] A. Berman and R.J. Plemmons. Nonnegative Matrices in the Mathematical Sciences (1979), Academic Press, San Diego, CA, reprinted by SIAM, Philadelphia, 1994

[5] L. Farina and S. Rinaldi, Positive Linear Systems: Theory and Applications. New York, Wiley-Interscience, 2000.

[6] A. Zaher, A. Traore and F. Thiery. Modélisation et contrôle d'une installation de cogénération, SFT, Société Française de Thermique, 2011.

[7] C. Gähler, M. Gwerder, R. Lamon and J. Tödtli. Optimal Control of Cogeneration Building Energy Systems, REHVA World Congress, Helsinki, 2007.

[8] W. M. Haddad, V. Chellaboina, and Q. Hui. Nonnegative and Compartmental Dynamical Systems. Princeton University Press, 2010.

[9] T. Kaczorek. Positive 1D and 2D Systems. Springer Verlag, UK, 2001.

[10] N. Kelly and I. Beausoleil-Morrison. Specifications for Modelling Fuel Cell and Combustion-Based Residential Cogeneration Devices within Whole-Building Simulation Programs, A Report of Subtask B of FC+COGEN-SIM, Annex 42 of the International Energy Agency Energy Conservation in Buildings and Community Systems Programme, October, 2007.

[11] D. G. Luenberger, *Introduction to Dynamic Systems: Theory, Models and Applications*. New York: John Wiley and Sons, 1979.

[12] M. Pirouti, J. Wu, J. Ekanayake and N. Jenkins, Dynamic Modelling and Control of a Direct-Combustion Biomass CHP Unit, Universities Power Engineering Conference (UPEC), 2010 45th International, Cardiff, Wales, 2010.

[13] A. Rantzer. Distributed control of positive systems. IEEE CDC Conference and European Control Conference (CDC-ECC), 2011.

[14] T. Schmid, D. Mehdi, G. Abba. Modeling and control of a thermal system. *IEEE Conference on Control Applications - CCA*, 1994

[15] M. Stemmann A. Rantzer. Temperature control of two interacting rooms with decoupled PI Control. *IFAC World Congress*, South Africa, 2014.

ISSN 2356-5608