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ROBUST ITERATIVE LEARNING CONTROLLER DESIGN FOR 2D UNCERTAIN LINEAR SYSTEMS SUBJECT TO EXTERNAL DISTURBANCES

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***Abstract*—This paper presents a robust iterative learning control (ILC) for a class of two dimensional (2D) linear systems with parametric uncertainty and considerable disturbances. The proposed control law is iteratively updated to guarantees the robust stability. Based on H_∞ setting, sufficient conditions for robust monotonic convergence of the proposed scheme are presented in terms of linear matrix inequality (LMI). A servo flexible example is presented in the end of this paper to demonstrate the effectiveness of the proposed learning algorithm.**

***Keywords*- uncertain systems iterative learning control, robust control, linear matrix inequality, 2D systems, H∞ setting and robust stability.**

1 INTRODUCTION

Iterative processes are a distinct class of two dimensional 2D systems of both theoretical and practical interest. These systems cannot be controlled and studied by direct application of existing techniques from standard 1D systems theory. The key unique feature of 2D systems is that the process dynamics depend on two independent variables propagating information in two independent directions [1, 2]. The study of the 2D systems is motivated by many applications such as repetitive processes [3, 4, 5, 6], control synthesis and processes theoretic problems and iterative learning [7, 8, 9, 10].

Iterative learning control uses knowledge processes from previous iteration of repeated motion to generate a feedforward control law to use on subsequent iterations and thereby aims to improve performance from pass to pass. It is clear that iterative learning processes have two dimensional 2D structure, where information propagation occurs along a given finite time interval (first direction t) and from iteration to iteration (second direction k).

The study and analysis of stability and robust stability of two dimensional continuous-discrete systems were investigated by Busłowicz, [11, 12, 13, 14], Bistritz [15, 16] and Xiao [17], the problem of monotonic convergence of 2D processes is also studied in [18]. These problems are solved based on several stability study approaches like H_∞ setting [18, 19], the performance weighting function [20] and the min-max method using the quadratic performance criterion [21].

Robust iterative learning control represents an important topic for controlling systems with parameters uncertainties. The synthesis of this type of control law is based on different approaches. The H_∞ approach based on resolution linear inequality matrix LMI problems offers the possibility to designing a new control law robust and effective used to improving the robust stability of 2D linear systems with considerable uncertainties in the parameters of matrix inputs. By using iterative learning controller the monotonic convergence and the systems stability are guaranteed and achieved after an esteemed number of iterations.

In this paper, we propose a robust and effective H_∞ and state feedforward controller design method for 2D state space systems with parameters uncertainties. The aim of this study is to develop new sufficient condition, based on LMI techniques, for robust iterative learning control law synthesis and stability analysis of 2D uncertain linear systems with disturbances.

The rest of this paper is organized as follows. The ILC problem is defined and the class of 2D uncertain systems is described in section II. In section III, sufficient conditions for robust stability and robust monotonic convergence, based on H_∞ setting with LMI techniques, are developed. A simulation results carried out on servo flexible system are presented in section IV. Finally, a discussion of the results and a conclusion are presented.

2 PROBLEM SETUP

The systems considered in this paper are described by two dimensional uncertain linear state space models with nonzero constant initial error and parametric uncertainty in the system:

$$\begin{cases} \dot{x}_{(k,t)} = (A+\Delta A)x_{(k,t)} + (B+\Delta B)u_{(k,t)} + (H+\Delta H)w_{(k,t)} \\ y_{(k,t)} = Cx_{(k,t)} \end{cases} \quad (1)$$

Where $x_k(t) \in R^n$ is the state vector, $y_k(t) \in R^n$ is the output, $u_k(t) \in R^m$ is the control input signal, $w_k(t) \in R^m$ is the disturbance, $A \in R^{n*n}$ is the constant matrix, $B \in R^{n*m}$ is the gain matrix of control input, $C \in R^{m*n}$ is the gain matrix of output, $H \in R^{n*m}$ is the gain matrix of disturbance input and ΔA, ΔB and ΔH represent admissible uncertainties. $k \geq 0$ denotes the number of iteration and $t \in [0,T]$. The boundary condition defined by $x(0) = x_0$.

The uncertainties matrices ΔA, ΔB and ΔH are supposed verifying the following assumption:

$$[\Delta A \quad \Delta B \quad \Delta H] = H_1 F [E_1 \quad E_2 \quad E_3] \quad (2)$$

where H_1, E_1, E_2 *and* E_3 are known constant matrices of compatible dimensions. F is unknown matrix with constant entries and satisfies

$$F^T F \leq I \quad (3)$$

Let us consider the reference model defined by a state space model:

$$\begin{cases} \dot{x}_d(t) = Ax_d(t) + Bu_d(t) \\ y_d(t) = Cx_d(t) \end{cases} \quad (4)$$

Where $x_d(t) \in R^n$, $u_d(t) \in R^m$ and $y_d(t) \in R^n$ represent respectively the reference state vector, the reference control input and the reference output.

The resetting condition is satisfied at each trial i.e. $x_d(0) = 0$, where $x_d(0)$ is the initial state of the referenced model.

A class of two dimensional linear uncertain systems with parametric uncertainty in the system and nonzero constant initial error is studied here. The H_∞ norm based on linear matrix inequality LMIs techniques is presented, in this paper, to design a new iterative algorithm to reduce the error from trial to trial and eliminate the uncertainty from the system. The monotonic convergence and the robust stability of 2D systems are guaranteed by using the proposed scheme. Our goal is to design and synthesis a new control law based on iterative learning control capable to drive the system described by (1) to follow the reference model described by (4) with zero error. The errors trajectory must decreases from iteration to iteration until becomes zero.

3 ROBUST STABILITY ANALYSIS

We present in this section, the analysis and synthesis of new robust iterative learning control for 2D uncertain linear systems with considerable disturbances described by (1). Based on the state space model description of the systems dynamics, the sufficient conditions which guarantee the robust stability of the system and the robust monotonic convergence is developed in this section in terms of the feasibility of LMIs.

For linear iterative processes of the form considered in the system (1), the general robust iterative learning control is described by the following structure:

$$\begin{cases} u_k(t) = v_{1,k}(t) + v_{2,k}(t) \\ v_{1,k}(t) = u_d(t) + K_{rob} e_k(t) \\ v_{2,k+1}(t) = v_{2,k}(t) + K_P ey_k(t) \end{cases} \quad (5)$$

The learning rules $v_{1,k}(t)$ and $v_{2,k}(t)$ represent respectively the robust control and the iterative learning control that is iteratively updated, where K_{rob} and K_P represent the learning gains matrix and $v_{2,0} = 0$.

We define the tracking error model as follows:

$$\begin{cases} e_k(t) = x_d(t) - x_k(t) \\ ey_k(t) = y_d(t) - y_k(t) \end{cases} \quad (6)$$

After substituting (1) into (4) and integrating the control law (5), the output error becomes:

$$ey_k(t) = Ce_k(t) \quad (7)$$

Let consider the following learning state variable:

$$\eta_{k+1} = \int_0^t x_{k+1}dt - \int_0^t x_k dt \quad (8)$$

With the help of the equality (1) and integrating the control law (5), we develop the new state variable described by the following expression:

$$\dot{\eta}_{k+1} = (A+\Delta A)\eta_{k+1} + (B+\Delta B)\tilde{u}_{k+1} + (H+\Delta H)\tilde{w}_{k+1} \quad (9)$$

Proof:

$$\dot{\eta}_{k+1} = \int_0^t \dot{x}_{k+1}dt - \int_0^t \dot{x}_k dt$$

$$= \int_0^t ((A+\Delta A)x_{k+1} + (B+\Delta B)u_{k+1} + (H+\Delta H)w_{k+1})dt -$$

$$\int_0^t ((A+\Delta A)x_k + (B+\Delta B)u_k + (H+\Delta H)w_k)dt$$

$$= (A+\Delta A)\int_0^t (x_{k+1} - x_k)dt + (B+\Delta B)\int_0^t (K_{rob}(e_{k+1} - e_k) + K_P ey_k)dt$$

$$+(H+\Delta H)\int_0^t (w_{k+1} - w_k)dt$$

$$= ((A+\Delta A) - (B+\Delta B)K_{rob})\int_0^t (x_{k+1} - x_k)dt +$$

$$(B+\Delta B)K_P\int_0^t ey_k dt + (H+\Delta H)\int_0^t (w_{k+1} - w_k)dt$$

$$= ((A+\Delta A) - (B+\Delta B)K_{rob})\eta_{k+1} + (H+\Delta H)\tilde{w}_{k+1}$$

$$+(B+\Delta B)K_P\tilde{e}y_k$$

$$= (A+\Delta A)\eta_{k+1} + (B+\Delta B)\tilde{u}_{k+1} + (H+\Delta H)\tilde{w}_{k+1}$$

Where:

$$\tilde{w}_{k+1} = \int_0^t (w_{k+1} - w_k)dt$$

$$\tilde{u}_{k+1} = -K_{rob}\eta_{k+1} + K_P\tilde{e}y_k$$

$$\tilde{e}y_k = \int_0^t ey_k dt$$

The error, at the iteration number k+1, is defined as follows:

$$\begin{aligned} ey_{k+1} &= -C(A+\Delta A)\eta_{k+1} - C(B+\Delta B)\tilde{u}_{k+1} + ey_k \\ &-C(H+\Delta H)\tilde{w}_{k+1} \end{aligned} \quad (10)$$

Proof:

From the equalities (7) and (8):

$$ey_{k+1} - ey_k = Ce_{k+1} - Ce_k$$

$$= -C(x_{k+1} - x_k)$$

$$= -C\dot{\eta}_{k+1}$$

Replacing $\dot{\eta}_{k+1}$ by their expression in the next equality, we get the equality (10).

From the equalities (9) and (10), we considered the new 2D uncertain linear system described by the following state representation:

$$\begin{bmatrix} \dot{\eta}_{k+1} \\ ey_{k+1} \end{bmatrix} = \left(\begin{bmatrix} A & B_0 \\ C_0 & D_0 \end{bmatrix} + \begin{bmatrix} \Delta A & 0 \\ \Delta C_0 & 0 \end{bmatrix} \right) \begin{bmatrix} \eta_{k+1} \\ ey_k \end{bmatrix} + \left(\begin{bmatrix} B \\ D \end{bmatrix} + \begin{bmatrix} \Delta B \\ \Delta D \end{bmatrix} \right) \tilde{u}_{k+1} + \left(\begin{bmatrix} B_{11} \\ D_{11} \end{bmatrix} + \begin{bmatrix} \Delta B_{11} \\ \Delta D_{11} \end{bmatrix} \right) \tilde{w}_{k+1} \quad (11)$$

Where:

$B_0 = 0$, $B_{11} = H$, $C_0 = -CA$, $D = -CB$, $D_0 = I$, $D_{11} = -CH$, $\Delta B_{11} = \Delta H$, $\Delta C_0 = -C\Delta A$, $\Delta D = -C\Delta B$ and $\Delta D_{11} = -C\Delta H$.

Based on (2) the induced uncertainties in the representation (11) verify the following condition:

$$\begin{bmatrix} \Delta A & \Delta B & \Delta B_{11} \\ \Delta C_0 & \Delta D & \Delta D_{11} \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} \quad (12)$$

Where $H_2 = -CH_1$

It is clear that the 2D system dynamics represented in (11) are affected by disturbances and uncertainties. The principal goal in this approach is the design of a robust gain

K_{rob} and a P type iterative learning gain K_P. These gains guarantee the system stability and the monotonic convergence while satisfying the H_∞ constraint.

To show stability of systems described by (11), we will require a Lyapunov function interpretation where the variable function is taken to be:

$$V(k,t) = \eta_{k+1}^T(t) P_1 \eta_{k+1}(t) + ey_{k+1}^T(t) P_2 ey_{k+1}(t) \qquad (13)$$

With $P_1 \succ 0$ and $P_2 \succ 0$.

It is now routine to conclude that stability along the pass holds if $\Delta V(k,t) \prec 0$.

Definition 1: The 2D linear iterative system defined in (11) is said to have H_∞ disturbance norm bound $\gamma \succ 0$ if the robust stability is guaranteed along the pass and the induced norm between the output and the disturbance input is bounded by γ.

Theorem 1: Suppose that a robust control law described by (5) is applied to a 2D linear iterative system of the form (11), with uncertainties form modeled by (3) and (12). Then, the resulting system is stable along the pass for all tolerable uncertainties and has H_∞ norm bound $\gamma \succ 0$ if there exist matrices $w_1 \succ 0, w_2 \succ 0$, N_1 and a scalar $\varepsilon \succ 0$ such that the LMI presented in (14) holds:

$$\begin{bmatrix} \varphi_{11} & * & * & * & * & * & * & * & * & * \\ \varphi_{21} & \varphi_{22} & * & * & * & * & * & * & * & * \\ w_2 & 0 & -w_2 & * & * & * & * & * & * & * \\ -K_{Piter}^T B^T C^T & K_{Piter}^T B^T & 0 & -I & * & * & * & * & * & * \\ -H^T C^T & H^T & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & w_2 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & I & 0 & 0 & -I & * & * & * \\ 0 & \varphi & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * \\ 0 & 0 & 0 & E_2 K_{Piter} & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & 0 & E_3 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} \prec 0 \qquad (14)$$

Where:

$$\varphi_{11} = -w_2 + 3\varepsilon H_2 H_2^T$$
$$\varphi_{21} = -w_1 A^T C^T + N_1^T B^T C^T + 3\varepsilon H_1 H_2^T$$
$$\varphi_{22} = w_1 A^T + A w_1 - N_1^T B^T - B N_1 + 3\varepsilon H_1 H_1^T$$
$$\varphi = E_1 w_1 - E_2 N_1$$

If (14) holds, the robust control law K_{rob} is given by $N_1 w_1^{-1}$ and the iterative control law K_P are given directly from the resolution of the LMI.

Proof: introduced the associated Hamiltonian as:

$$\mathbb{H}(k,t) = \Delta V(k,t) + ey_k^T(t) ey_k(t) - \gamma^2 \tilde{w}_{k+1}^T(t) \tilde{w}_{k+1}(t) \qquad (15)$$

And it is simple to show that H_∞ disturbance attenuation is equivalent to:

$$\mathbb{H}(k,t) \prec 0$$

We can write:

$$\mathbb{H} = \breve{X}^T \Phi \breve{X} \qquad (16)$$

Where:

$$\breve{X} = \begin{bmatrix} \eta_{k+1}(t) & ey_k(t) & \tilde{e}y_k(t) & \varpi_{k+1}(t) \end{bmatrix}^T$$

$$\Phi = \begin{bmatrix} \hat{A}_1^T P + P\hat{A}_1 + \hat{A}_2^T \bar{S} \hat{A}_2 + \bar{L}^T \bar{L} + \bar{M}^T \bar{M} - R & \bar{P}\hat{B}_1 + \hat{A}_2^T \bar{S} \hat{B}_2 \\ \hat{B}_1^T \bar{P} + \hat{B}_2^T \bar{S} \hat{A}_2 & -\gamma^2 I + \hat{B}_2^T \bar{S} \hat{B}_2 \end{bmatrix} \qquad (17)$$

And

$$\bar{S} = \begin{bmatrix} P_4 & 0 & 0 \\ 0 & P_3 & 0 \\ 0 & 0 & P_2 \end{bmatrix} \quad \hat{B}_2 = \begin{bmatrix} 0 \\ 0 \\ D_{11} + \Delta D_{11} \end{bmatrix} \quad \hat{B}_1 = \begin{bmatrix} B_{11} + \Delta B_{11} \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \bar{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (C_0 + \Delta C_0) - (D + \Delta D)\mathrm{K}_{rob} & D_0 & (D + \Delta D) K_P \end{bmatrix}$$

$$\hat{A}_1 = \begin{bmatrix} (\mathrm{A} + \Delta A) - (B + \Delta B)\mathrm{K}_{rob} & B_0 & (B + \Delta B) K_P \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{L} = \begin{bmatrix} 0 & I & 0 \end{bmatrix} \quad \bar{M} = \begin{bmatrix} 0 & 0 & I \end{bmatrix}$$

Applying a three successive modified Schur lemma to the equality (17) followed by replacing the variables by theirs appropriates expressions in the result then pre and post multiply the result by $T = diag\{P_4^{-1}, P_3^{-1}, P_2^{-1}, P_1^{-1}, P_2^{-1}, I, I, I, I\}$ to eliminate the bilinearity.

Then setting $N_1 = K_{rob}P_1^{-1}$, $w_1 = P_1^{-1}$, $w_2 = P_2^{-1}$, $w_3 = P_3^{-1}$, $w_4 = P_4^{-1}$ in the result. Finally, noting that the result doesn't depend to w_3 and w_4 leads to:

$$\Psi = \begin{bmatrix} -w_2 & * & * & * & * & * & * \\ \alpha_1 & \alpha_2 & * & * & * & * & * \\ w_2 D_0^T & w_2 B_0^T & -w_2 & * & * & * & * \\ K_P{}^T D^T & K_P{}^T B^T & 0 & -I & * & * & * \\ D_{11}{}^T & B_{11}{}^T & 0 & 0 & -\gamma^2 I & * & * \\ 0 & 0 & w_2 & 0 & 0 & -I & * \\ 0 & 0 & 0 & I & 0 & 0 & -I \end{bmatrix} + \begin{bmatrix} 0 & * & * & * & * & * & * \\ \alpha_3 & \alpha_4 & * & * & * & * & * \\ w_2 \Delta D_0^T & w_2 \Delta B_0^T & 0 & * & * & * & * \\ K_P{}^T \Delta D^T & K_P{}^T \Delta B^T & 0 & 0 & * & * & * \\ \Delta D_{11}{}^T & \Delta B_{11}{}^T & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

Where: $\alpha_1 = w_1 C_0{}^T - N_1{}^T D^T$,
$\alpha_2 = w_1 A^T + A w_1 - N_1{}^T B^T - B N_1$, $\alpha_3 = w_1 \Delta C_0{}^T - N_1{}^T \Delta D^T$,
$\alpha_4 = w_1 \Delta A^T + \Delta A w_1 - N_1{}^T \Delta B^T - \Delta B N_1$

The second term in the above inequality can be written as:

$$\bar{H}\bar{F}\bar{E} + \bar{E}^T \bar{F}^T \bar{H}^T \quad (19)$$

Where:

$$\bar{H} = \begin{bmatrix} 0 & H_2 & 0 & H_2 & H_2 & 0 & 0 \\ 0 & H_1 & 0 & H_1 & H_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{F} = diag\{F, F, F, F, F, F, F\}$$

$$\bar{E} = diag\{0, E_1 w_1 - E_2 N_1, 0, E_2 K_P, E_3, 0, 0\}$$

Lemma 1: Let Σ_1 and Σ_2 be real matrices of appropriate dimensions. Then for any matrix F satisfying $F^T F \le I$ and a scalar $\varepsilon > 0$ the following inequality holds [22]:

$$\Sigma_1 F \Sigma_2 + \Sigma_2^T F^T \Sigma_1^T \le \varepsilon^{-1} \Sigma_1 \Sigma_1^T + \varepsilon \Sigma_2^T \Sigma_2 \quad (20)$$

An obvious application of lemma 1 followed by application of the Schur complement lemma and replacing the variables by theirs expression yields (14) and the proof is complete.

4 SIMULATION EXAMPLE

To prove the efficiency of our RILC approach we use the mechanical example represented by a train consisting of a four masses M_1, M2, M_3 and M_4 mutually connected by a spring of stiffness k_i and braked by a dynamic friction coefficient c_i [23].

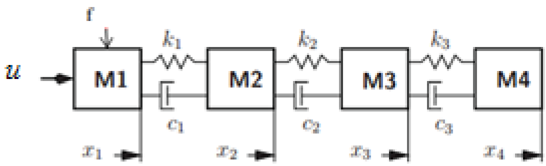

Fig. 1. System masses spring.

Where:
$m_1 = 1Kg$, $m_2 = m_3 = m_4 = 0.5Kg$,
$k_1 = k_2 = k_3 = 0.5Nm^{-1}$,
$c_1 = c_2 = c_3 = 0.5Nsm^{-1}$
This process is defined by the state space model described in the equality (1).

The state variable x is defined as follows:

$$x = (x_1 \quad \dot{x}_1 \quad x_2 \quad \dot{x}_2 \quad x_3 \quad \dot{x}_3 \quad x_4 \quad \dot{x}_4)^T$$

The desired input (u represented by u) and the disturbance represented as follows:

$$u = 2|\sin(2\pi t)|$$

$$f = 0.2|\sin(2\pi t)|$$

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{c_1+c_2}{m_2} & \frac{k_2}{m_2} & \frac{c_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{k_2}{m_3} & \frac{c_2}{m_3} & -\frac{k_2+k_3}{m_3} & -\frac{c_2+c_3}{m_3} & \frac{k_3}{m_3} & \frac{c_3}{m_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{k_3}{m_4} & \frac{c_3}{m_4} & -\frac{k_3}{m_4} & -\frac{c_3}{m_4} \end{bmatrix}$$

$$B=\begin{pmatrix} 0 & \frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T,$$

$$C=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H=\begin{pmatrix} 0 & \frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

Suppose that:

$$m_1 = m + \Delta m_1 = m(1+\xi)$$

$$\zeta = \xi / (1-\xi)$$

For $\xi = 0.1$ applying the decomposition procedure given by (12), we get $H_1 = \begin{bmatrix} 0 & -\frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$

$$H_2 = \begin{bmatrix} 0 & \frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$E_1 = \begin{bmatrix} -k_1 & -c_1 & k_1 & c_1 & 0 & 0 & 0 & 0 \end{bmatrix}$, $E_2 = -1$, $E_3 = -1$ and $F = \zeta$.

The simulation results are obtained for the initial state vector zero. The feasible solution of the LMI (14) is given by $\varepsilon = 0.2071$,

$$K_{rob} = \begin{bmatrix} -0.2269 & -0.0488 & 0.3324 & 0.2308 & -0.0664 & -0.1117 & -0.0391 & -0.0655 \end{bmatrix}$$

and $K_P = 0.4071$

Figure 2 and figure 4 show desired output trajectory (yd) and the output trajectory of the uncertain system (y) at the first iteration and at the last iteration, respectively. Figure 3 and figure 5 represents the errors trajectory of the uncertain at the first iteration and at the last iteration, respectively. Figure 6 shows the simulation results of proposed scheme to the uncertain system: outputs errors norm $\|e(t,k)\|_2$ and maximum outputs errors norm $\|e(t,k)\|$ versus iteration k.

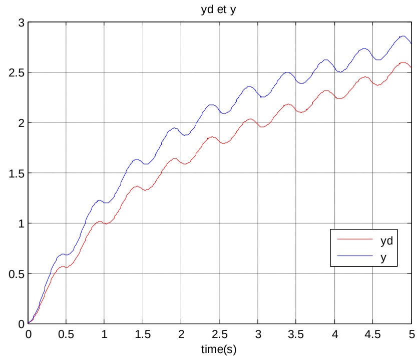

Fig.2. desired output trajectory (yd) and the output trajectory of the uncertain system (y) at the first iteration.

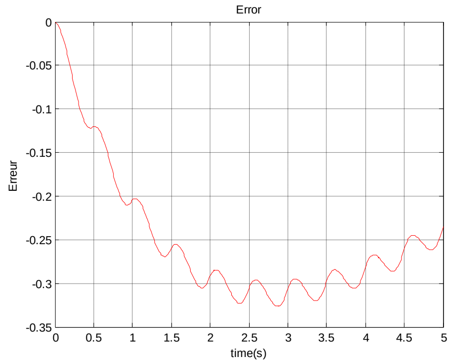

Fig.3. Error trajectory of the uncertain system at the first iteration.

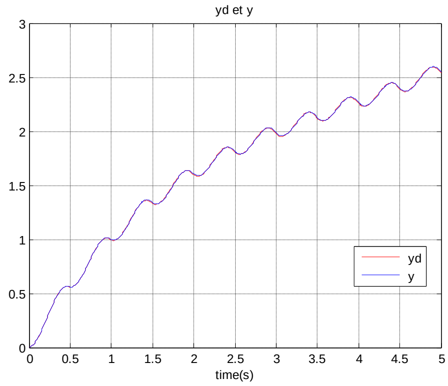

Fig.4. Desired output trajectory (yd) and the output trajectory of the uncertain system (y) and at the last iteration.

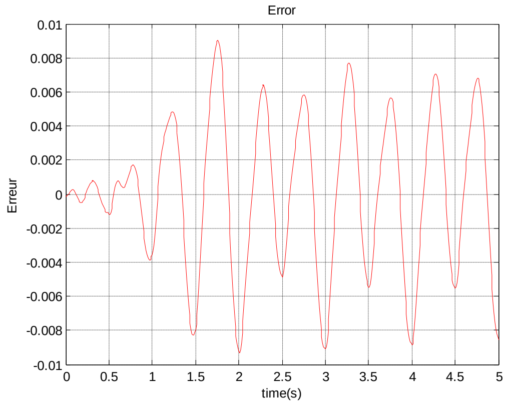

Fig.5. Error trajectory of the uncertain system at the iteration number 300.

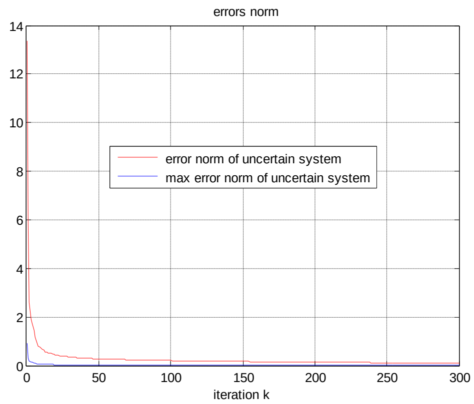

Fig.6. Simulation results of proposed scheme to the uncertain system: outputs errors norm $\|e(t,k)\|_2$ and maximum outputs errors norm $\|e(t,k)\|$ versus iteration k.

It is very clear that errors of the uncertain system decreases from iteration to iteration until becomes zero from the iteration number 20. The robust monotonic convergence is achieved and the stability of the system is demonstrated. The robust iterative learning control is designed well and it achieved the objective of the present approach. Our approach is fast comparing to others research work, in this example we can see that the convergence is achieved in the iteration 20.

5 CONCLUSION

A robust Monotonic convergence problem for a class of 2D linear systems with parametric uncertainty with non-zero constant initial error in the system is studied in this paper. Robust stability is successfully proved. Based on H infinity setting using the LMI techniques, a new robust iterative learning control is designed for uncertain linear systems with considerable disturbances. The sufficient conditions are given by the LMIs which can directly determine the learning gains of the proposed control law.

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