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On the Internal Model Control of Multivariable Linear Underactuated Systems: Effects of Initial Conditions

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***Abstract*—The Internal Model Control (IMC) design based on the use of a specific inverse model was extended to the case of multivariable linear underactuated systems. This work deals with the effect of initial conditions on the IMC of multivariable linear underactuated systems behavior. The simulation results will be presented in this paper to show that initial conditions do not affect the ability of this control approach to ensure stability, accuracy and disturbance rejection.**

***Keywords*— *Internal model control; Linear multivariable underactuated systems; Initial conditions; Stability; Accuracy; Disturbance rejection*.**

I. INTRODUCTION

Multivariable system control is extremely important in industrial applications, so it becomes one of the most important research domains.

The internal model control is one of the most interesting and powerful multivariable system control approaches. It is characterized by its robustness and simplicity.

IMC concept was proposed in 1982 for single-input single-output systems by Garcia and Morari and it was extended to multi-input multi-output systems in 1985 [3, 8, 9]. The study was extended to the case of multivariable underactuated systems where the number of system inputs is less than the number of system outputs. The realized research treats the effects of the control input on the IMC underactuated system evolution. The obtained results were very encouraging which led us to treat the case of the initial conditions' effect.

We propose in this paper the IMC designed for linear multivariable underactuated systems such that initial conditions are taken into account; we present the effectiveness of this approach to ensure stability and preserve system performance despite the presence of external disturbances.

The realized work will be described in this paper through four sections. The second section presents a brief description of the basic IMC structure, the third section describes the proposed approach and the final section presents the simulation results.

II. IMC STRUCTURE DESCRIPTION

The internal model control is a robust control structure using the feedback concept.

The IMC basic configuration shown in Fig. 1, includes an internal model which is an explicit model of the plant, a controller which can be chosen the inverse of the process model and, if necessary, a filter [1, 7].

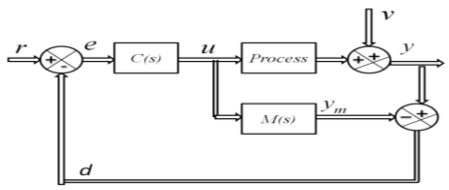

Fig. 1. IMC basic structure.

As shown in Fig. 1, *C(s)* is the IMC controller, *M(s)* the chosen model, *r* the reference signal, *v* the disturbance signal affecting the system outputs and *u* is the control input signal. *y* is the process output signal, y_m the model output signal and *d* is the difference between *y* and y_m. *d* represents also the disturbances effect and modeling errors. It is compared to the reference signal *r* to generate the controller input signal *e* [4, 9].

The IMC design is based on a specific principle of inversion which represents the main problem of this control approach. In fact, the realization of the direct model's inverse is difficult for many physical systems. This difficulty is due to the denominator order which is usually greater than the numerator one on the model expression or to the presence of unstable zeros or/and time delay [7, 5, 6, 1].

III. IMC FOR MULTIVARIABLE UNDERACTUATED SYSTEMS

The objective of the multivariable systems control is to manipulate simultaneously several input channels in order to obtain a desirable behavior of several output variables. Indeed, the system output parameters have to reach its input parameters despite the presence of non-controllable disturbances affecting the system [2, 9].

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The systems where the number of inputs is less than the outputs one are called underactuated systems. The IMC design defined for these systems was proposed in [9]; it's illustrated in Fig. 2

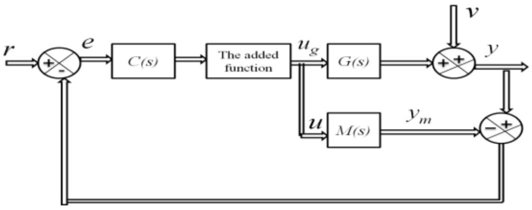

Fig. 2. IMC structure for multivariable underactuated systems.

where *G(s)* is the system transfer matrix and *M(s)* is the descriptive matrix of the model. *M* is chosen close to G. The proposed controller *C(s)* is obtained using the inversion method proposed in [7, 1] as shown in Fig. 3

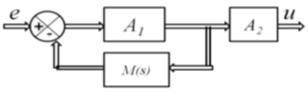

Fig. 3. The proposed IMC controller.

where A_1 is a matrix gain chosen to ensure the stability condition and A_2 is a matrix gain used to compensate the static error [7, 1, 2, 9]. *C(s)* and A_2 expressions are given by (1) and (2) [7, 1, 2, 9]

$$C(s) = A_1(I + A_1 M(s))^{-1} \tag{1}$$

$$A_2 = (I + A_1 M(0))(A_1 M(0))^{-1} \tag{2}$$

where *M(0)* is the static matrix gain of the chosen model *M*.

The stability of the system depends of both model *M* and controller *C*. The system is stable if and only if each block of the IMC structure is stable in open loop. That means the denominator of each transfer function of the model matrix *M* should be a Hurwitz polynomial. To ensure the stability of the controller *C*, it's necessary to choose the matrix A_1 for which the characteristic roots of the denominator of the blocs C_{ij} have to be negative real part [1, 7, 2, 9].

The inversion problem requires that the matrix *M* must be square [7, 1, 2, 9], but the underactuated system's transfer matrix *G* expressed by (4) is of dimension ($n \times m$), where *n* is the system's outputs number, *m* is the system's inputs number and *m* is less than *n*. The realized research in [9] proposes a solution to solve this problem by adding ($n \times (n-m)$) transfer functions to the matrix *M* in order to make it a square matrix of dimension ($n \times n$) (expressed by (5)), then a new function used to eliminate the ($n-m$) excess control inputs is added to the basic IMC structure as shown in Fig. 2. The following vector represents the obtained control input vector after inserting the added function [9]

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \\ \hline u_{m+1} = 0 \\ \vdots \\ u_n = 0 \end{bmatrix} \tag{3}$$

with $u_g = [u_1 \cdots u_m]^T$ is the vector of the control inputs acting on the process.

$$G(s) = \begin{bmatrix} G_{11}(s) & \cdots & G_{1m}(s) \\ \vdots & & \vdots \\ G_{n1}(s) & \cdots & G_{nm}(s) \end{bmatrix} \tag{4}$$

$$M(s) = \underbrace{\begin{bmatrix} M_{11}(s) & \cdots & M_{1m}(s) \\ \vdots & & \vdots \\ M_{n1}(s) & \cdots & M_{nm}(s) \end{bmatrix}}_{\substack{\text{Initial bloc} \\ (n \times m)}} \underbrace{\begin{bmatrix} M_{1m+1}(s) & \cdots & M_{1n}(s) \\ \vdots & & \vdots \\ M_{nm+1}(s) & \cdots & M_{nn}(s) \end{bmatrix}}_{\substack{\text{added bloc} \\ (n \times (n-m))}} \tag{5}$$

The study developed in [9] treated the influence of the control input signal on the system behavior; in fact, the excess control inputs acting on the added bloc of the matrix *M* were eliminated as was explained previously. The obtained results were very encouraging which led us to treat the case of the initial conditions in order to show their effect on the system evolution.

The system transfer matrix representation developed in [9] do not reveal what will happen if the system is not initially relaxed [10]. This fact led us to translate the results obtained in [9] to the state-space representation. Therefore, the system can be described by the following state-space

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \\ x(t_0) = x_0 \end{cases} \tag{6}$$

where

- $x(t)$: The system state vector of dimension ($p \times 1$);
- $y(t)$: The system output vector of dimension ($q \times 1$);
- $u(t)$: The control input vector of dimension ($l \times 1$);
- A : The state matrix of dimension ($p \times p$);
- B : The control matrix of dimension ($p \times q$);
- C : The observation matrix of dimension ($q \times p$);
- D : The direct transmission matrix of dimension ($q \times l$);
- x_0 : The initial state vector of dimension ($p \times 1$).

IV. APPLICATION(EFFECTS OF INITIAL CONDITIONS)

In order to show the effects of the initial state vector on the behavior of the IMC underactuated systems, let's consider the case of the one input/two outputs system proposed in [9]. It's represented by the following transfer matrix *G(s)* and modeled by the following matrix *M(s)* [9].

$$G(s) = \begin{pmatrix} \dfrac{2s+2}{s^2+4s+2} \\ \dfrac{2s+1}{3s^2+3s+1} \end{pmatrix} \qquad M(s) = \begin{pmatrix} \dfrac{s+2}{s^2+3s+2} & \dfrac{0.5}{0.5s+1} \\ \dfrac{s+1}{4s^2+3s+2} & \dfrac{0.75}{0.25s+1} \end{pmatrix} \tag{7}$$

With $y = [y_1 \quad y_2]^T$ is the system output vector and the reference vector *r* is chosen step of amplitude 1.

The case of disturbed system is considered in order to show a significant improvement of the velocity, the accuracy and the disturbance rejection capability of the proposed IMC configuration despite the non-null system's initial conditions. The disturbance vector expression is given by (8).

$$V(s)=\begin{bmatrix} \frac{e^{-30s}}{s} & \frac{e^{-30s}}{s} \end{bmatrix}^T \tag{8}$$

As we noted on the previous section, we convert the system transfer matrix representation into an equivalent state-space representation. The state-space representations (in the canonical form) of the process and its model are given by (9) and (10).

$$A_G=\begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-1}{3} & -1 \end{pmatrix} \quad B_G=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad C_G=\begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix} \tag{9}$$

$$A_M=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.5 & -0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 \end{pmatrix} \quad B_M=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_M=\begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0.25 & 3 \end{pmatrix} \tag{10}$$

In order to test the proposed approach, several measures of the model's initial state vector will be taken in the following.

- The first example

The model's initial state vector denoted by x_{0_M} is given by (11). The initial conditions of the model implemented in the controller (shown in Fig. 3) are assumed null.

$$x_{0_M}=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{11}$$

The two system outputs y_1 and y_2 are shown in Fig. 4 and Fig. 5.

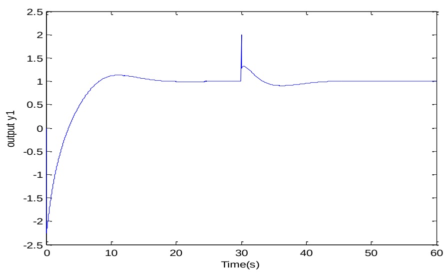

Fig. 4. The output y_1 of the system.

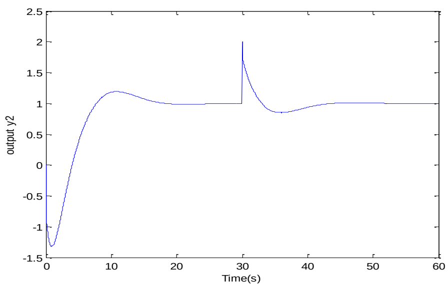

Fig. 5. The output y_2 of the system.

- The second example

The initial state vector of the model implemented in the controller (denoted by x_{0_C}) is given by (12). The initial conditions of the system model are considered null.

$$x_{0_C}=\begin{bmatrix} -1 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}^T \tag{12}$$

The two system outputs y_1 and y_2 are shown in Fig. 6 and Fig. 7.

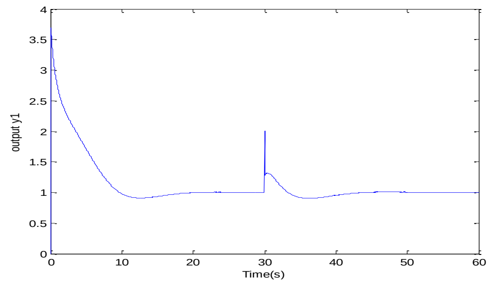

Fig. 6. The output y_1 of the system.

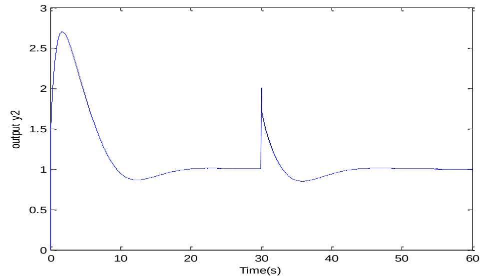

Fig. 7. The output y_2 of the system.

- The third example

The two initial state vectors x_{0_M} and x_{0_C} are given by (13).

$$x_{0_M}=\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}^T \quad x_{0_C}=\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}^T \tag{13}$$

The two system outputs are shown in Fig. 8 and Fig. 9.

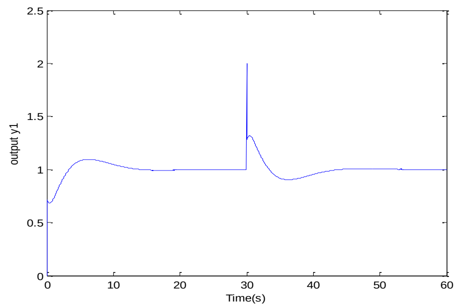

Fig. 8. The output y_1 of the system.

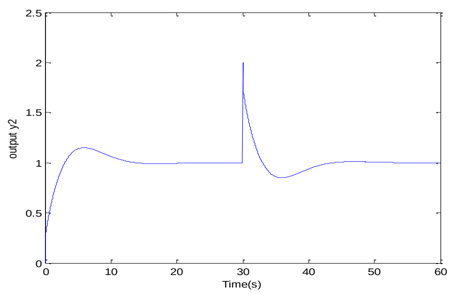

Fig. 9. The output y_2 of the system.

• The fourth example

The two initial state vectors are given by (14).

$$x_{0_M} = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}^T \qquad x_{0_C} = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}^T \quad (14)$$

The two system outputs are shown in Fig. 10 and Fig. 11.

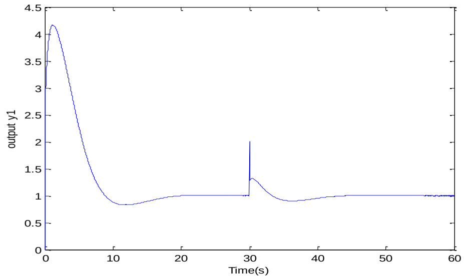

Fig. 10. The output y_1 of the system.

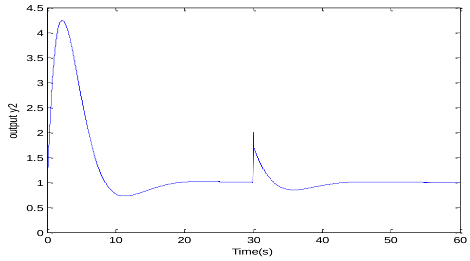

Fig. 11. The output y_2 of the system.

• The fifth example

The two initial state vectors are given by (15).

$$x_{0_M} = x_{0_C} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \qquad (15)$$

The two system outputs are shown in Fig. 12 and Fig. 13.

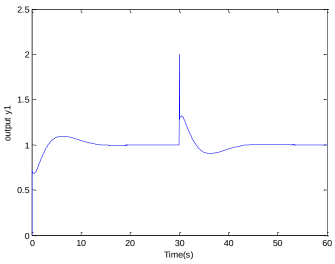

Fig. 12. The system output y_1.

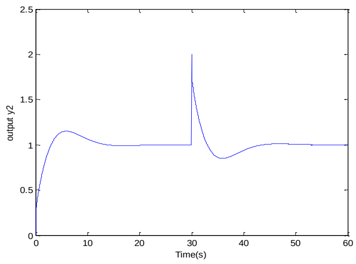

Fig. 13. The system output y_2.

The simulation results show that the initial conditions affect only the transient region of the output signal. The system outputs reach the reference signal and the disturbance was rapidly rejected despite the disturbance vector which attacks the system outputs directly, proving the robustness of the proposed control approach.

V. Conclusion

This paper presents a new approach for IMC of linear multivariable underactuated systems. The realized research deals with the effect of the initial conditions on the behavior of the system. The obtained simulation results prove the accuracy and the rapid disturbance rejection capability of the IMC of underactuated systems even in the case where the initial conditions of the system are taken into account.

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