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Performance Evaluation of Proportional Navigation Homing Guidance Law

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Abstract—**Homing tactical missile is one of the most important guided weapons used for destroying the air targets. Proportional navigation (PN) had been proved to be an optimal guidance strategy for homing guided missiles. Due to its ease in implementation and its effectiveness against maneuvering and non-maneuvering targets, PN is receiving great attentions of many researches.**
This paper presents an investigation study using MATLAB for PN under different scenarios of missile-target engagement. The basic 2-D geometric of PN is first established. Then, modeling and simulating PN under the main sources of errors which include the missile initial heading error and target maneuver is carried out. Finally, the idea of time varying navigation gain (TVNG) and its impact on the enhancement of PN specially in the presence of noise in the tracking system will be introduced.

***Keywords*— guidance system, homing guidance, proportional navigation.**

I. INTRODUCTION

Missile guidance concerns the method by which the missile receives its commands to move along a certain flight path towards its target. Some missiles generate these commands onboard (homing guidance), while others receive these commands from external control site (command guidance).
Many guided missiles employ some version of PN as the guidance law during the terminal homing phase. Surface-to-air, air-to-air, and air-to-surface missile engagements, as well as space applications (including rendezvous), use PN in one form or another as a guidance law[1]. The Lark missile, which had its first successful test in December 1950, was the first missile to use proportional navigation. Since that time proportional navigation guidance has been used in virtually all of the world's tactical radar, infrared (IR), and television (TV) guided missiles [2].A major advantage of PN, is its relative simplicity of implementation in practical systems. For implementation, it requires low levels of information input regarding the target characteristics (including motion) compared with many other more elaborate schemes, thus simplifying on board sensor requirements and improving reliability and robustness. The scheme is based entirely on the instantaneous direction of the target relative to the pursuer in space, and its first derivative with respect to time[3],[4]. PN guidance law issues acceleration commands, perpendicular to the instantaneous missile-target line of sight, which are proportional to line of sight rate and closing velocity. In tactical radar homing missiles using proportional navigation guidance, the seeker provides an effective measurement of the line of sight rate, and a Doppler radar provides closing velocity information. In tactical IR missile applications of proportional navigation guidance, the line of sight rate is measured, whereas the closing velocity, required by the guidance law, is guesstimated. In tactical end atmospheric missiles, proportional navigation guidance commands are usually implemented by moving fins or other control surfaces to obtain the required lift. Exo atmospheric strategic interceptors use thrust vector control, lateral divert engines, or achieve the desired acceleration levels[2].

II. TWO DIMENSIONAL SIMULATION OF PROPORTIONAL NAVIGATION

For simplicity, a two dimensional model is considered. The missile and target are assumed mass points with velocities V_M and V_T as shown in Fig 1.

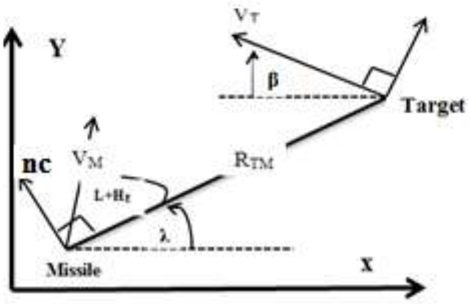

Fig. 1 Two dimensional missile-target engagement geometry

From Fig. 1 we can see that the missile, with velocity magnitude V_M ,is heading at an angle of (L + HE) with respect to the line of sight (LOS).
The angle L is known as the missile lead angle. The lead angle is the theoretically correct angle for the missile to be on collision triangle with the target. In Fig. 1 the imaginary line connecting the missile and. target is known as the line of sight. The line of sight makes an angle of λ with respect to the fixed reference, and the length of the line of sight (instantaneous separation between missile and target) is a range denoted R_{TM}. From a guidance point of view, it is required to make the range between missile and target at the expected intercept time as small as possible. The point of closest approach of the missile and target as the miss distance.

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In the engagement model of Fig. 1 the target can maneuver evasively with acceleration magnitude perpendicular to the target velocity vector, the angular velocity of the target can be expressed as

$$\dot{\beta} = \frac{n_T}{V_T} \quad (1)$$

Where V_T is the magnitude of the target velocity.

The components of the target velocity vector in the Earth or inertial coordinate system are

$$V_{T1} = -V_T cos\beta \quad (2)$$

and

$$V_{T2} = V_T sin\beta \quad (3)$$

The differential equations for the components of the target position are given by

$$\dot{R}_{TX} = V_{TX} \quad (4)$$

and

$$\dot{R}_{TY} = V_{TY} \quad (5)$$

Similarly, the missile velocity differential equation are given by

$$\dot{V}_{MX} = A_{MX} \quad (6)$$

and

$$\dot{V}_{MY} = A_{MY} \quad (7)$$

The differential equations for the components of the missile position are given by

$$\dot{R}_{MX} = V_{MX} \quad (8)$$

and

$$\dot{R}_{MY} = V_{MY} \quad (9)$$

Where A_{MY}, A_{MY}, V_{MX} and V_{MY} are the accelerations and velocity components in the earth coordinate .system. The components of the relative missile-target separation are

$$R_{TM} = R_{TX} - R_{MX} \quad (10)$$

and

$$R_{TMY} = R_{TY} - R_{MY} \quad (11)$$

The relative velocity components in Earth coordinates to be

$$V_{TM1} = V_{TX} - V_{MX} \quad (12)$$

and

$$V_{TMY} = V_{TY} - V_{MY} \quad (13)$$

The closing velocity V_C is defined as the negative rate of change of the distance from the missile to the target or

$$V_C = -\dot{R}_{TM} \quad (14)$$

Therefore at the end of the engagement, when the missile and target are in closest proximity, the sign of $V_C$will change. In other words, in calculus we know that the closing velocity will be zero when .minimum (i.e., the function is either minimum or maximum when its derivative is zero): The desired acceleration command n_C which is derived from the proportional navigation guidance law, is perpendicular to the instantaneous line of sight.

The LOS angle can be found, using trigonometry, in terms of the relative separation components as

$$\lambda = \tan^{-1}\frac{R_{TMY}}{R_{TMX}} \quad (15)$$

The relative velocity components in Earth coordinates are

$$V_{TMX} = V_{TX} - V_{MX} \quad (16)$$

and

$$V_{TMY} = V_{TY} - V_{MY} \quad (17)$$

The LOS rate can be obtained by direct differentiation of the expression for LOS angle. After some algebra we obtain the expression for the LOS rate to be

$$\dot{\lambda} = \frac{R_{TMX}V_{TMY} - R_{TMY}V_{TMX}}{R_{TM}^2} \quad (18)$$

Where

$$R_{TM} = (R_{TMX}^2 + R_{TMY}^2)^{0.5} \quad (19)$$

The closing velocity is defined as the negative rate of change of the missile target separation, it can be obtained by differentiating the preceding equation, yielding

$$V_C = -\dot{R}_{TM} = \frac{-(R_{TMX}\,V_{TMX} - R_{TMY}\,V_{TMY})}{(R_{TM})} \quad (20)$$

The magnitude of the missile guidance command n_C can then be found expressed as

$$n_C = V_C\,\acute{N}\,\dot{\lambda} \quad (21)$$

Since the acceleration command is perpendicular to the instantaneous line of sight, the missile acceleration components in Earth coordinates can be found by trigonometry using the angular definitions from Fig. 1.

The missile acceleration components are.

$$A_{MX} = -n_C \sin(\Theta + H_E) \quad (22)$$

and

$$A_{MY} = n_C \cos(\Theta + H_E) \quad (23)$$

Where Θ is given by

$$\Theta = \lambda + L \quad (24)$$

And H_E is heading angle which represents the initial deviation of the missile from the collision triangle.

A missile employing proportional navigation guidance is not fired at the target but is fired in a direction to lead the target. The initial angle of the missile velocity vector with respect to the LOS is known as the missile lead angle *L*. The theoretical missile lead angle can be found by application of the law of sins , yielding

$$L = \sin^{-1}\frac{V_T\sin(\beta+\lambda)}{V_M} \quad (25)$$

The nonlinear system of equations given by eq's 1 to 25 represents the 2- dimensional kinematics model of a homing guided missile. Numerical integration is used to numerically solve this system.

III. SIMULATION RESULTS

A MATLAB code was set up for a two-dimensional missile-target engagement simulation using the differential equations derived in the previous section. The simulation is to understand the effectiveness of proportional navigation, it is best to simulate the guidance law and test its properties under a variety of circumstances. Inputs are the initial location of the missile and target, speeds, flight time, and effective navigation ratio. The two error sources considered here will be the missile initial heading error and the target manoeuvre.

A. Effect Of Missile Initial Heading Error

In Fig.2 sample trajectories for different navigation ratios are depicted. We can from the figure that initially the missile is flying in wrong direction because of the initially heading error. Gradually the guidance law forces the missile to home on to the target. The larger navigation ratio enable the missile to remove the initial heading error more rapidly, thus causing a much tighter trajectory. in all the cases the missile hits the target(zero miss distance with simulation). The resultant missile demanded acceleration is displayed in Fig. 3. The quicker removal of heading error with higher navigation ratio(N=5) results in higher missile demanded acceleration at the beginning of flight and lower acceleration near the of flight.

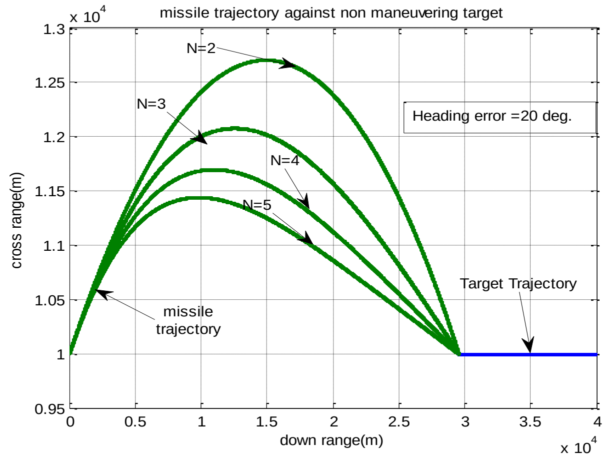

Fig. 2 Missile trajectories using PN with initial heading error

B. *Effect Of Target Manoeuvre*

In case of if the target manoeuvring is considered, the missile and target are initially on a collision course. At time t=0 the target initiates a lateral acceleration normal to the target collision flight path, Fig 4 displays the trajectories for missile guided PN for different effective navigation ratios N. It is clear that the higher navigation ratio causes the missile to lead the target slightly more than the lower one. In all the cases the PN enabled the missile to hit the target. Fig. 5 shows the resultant missile demanded acceleration required by the missile to hit manoeuvring target, we can see that the higher effective navigation ratio requires less acceleration capability of the missile, and the peak acceleration required by the missile to hit the target is significantly higher than manoeuvring level of the target.

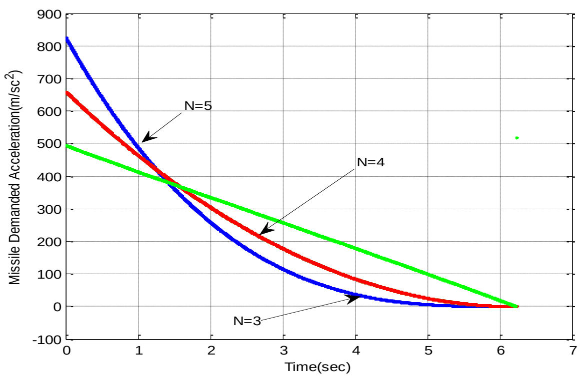

Fig. 3 Missile demanded acceleration using PN to eliminate initial heading error

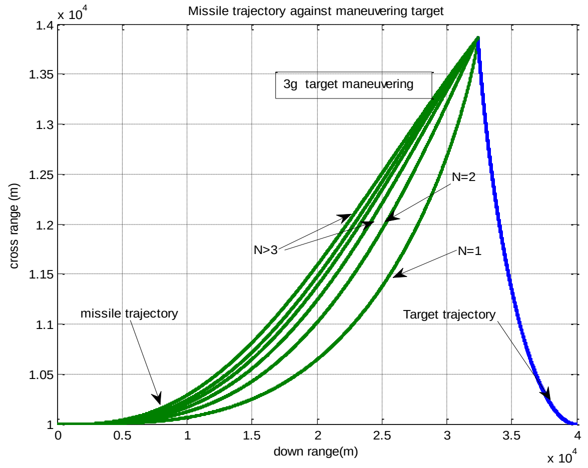

Fig. 4 Missile trajectories using PN against manoeuvring target

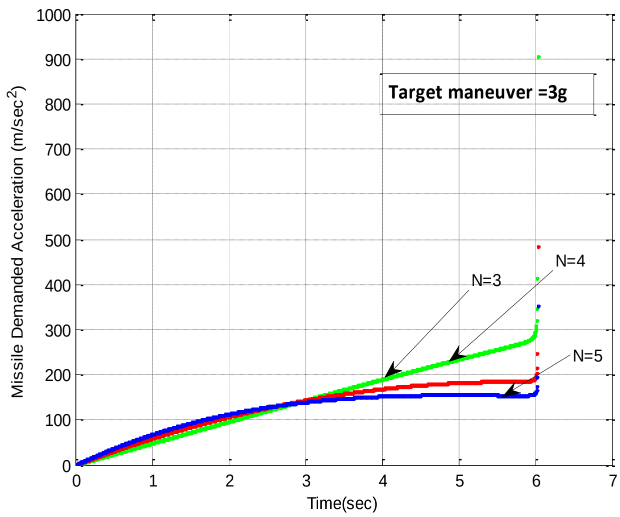

Fig. 5 Missile demanded acceleration using PN against manoeuvring target

IV. PN WITH TIME VARYING NAVIGATION GAIN(TVNG)

In the previous section, the analysis of PN showed that the large navigation gain leads to faster correction of the guidance errors which increases the demanded acceleration initially. This may be considered as disadvantage specially if the missile acceleration limitation is expected. On the other hand, near the engagement end, the large value of navigation gain results in small miss distance and better guidance accuracy. From the point of view of missile target separation and the missile demanded acceleration, it is better to start with lower navigation gain and making it increases rapidly as the missile approaches the target. In addition to that the practical utilization of the guidance system. For example, in radar homing tactical missiles the main function of the seeker is the measurement of LOS rate signal, which is then used to implement the PN guidance signal. The measurement of this signal specially for large missile-target separation(the target signal is relatively weak) is usually not perfect, it is corrupted with noise which reduces the efficiency and the performance of the whole system and hence, decreases the probability of hitting.

In this section we will introduce the TVNG to reduce the effect of noise on the performance of PN. The variation of the navigation gain with time, is considered to be linear, convex, and concave as shown in Fig. 6. The missile demanded acceleration for PN with TVNG against manoeuvring target are displayed in Fig. 7. As expected the constant gain case yields maximum acceleration early in flight and the TVNG results in maximum acceleration near the end of flight. The advantages of this variable gain method will appear in the next section in the presence of noise.

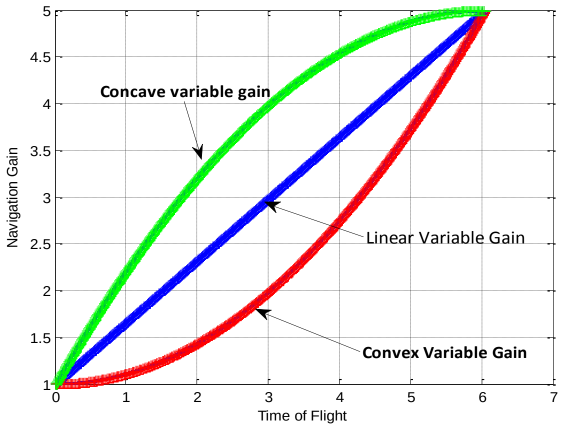

Fig. 6 Navigation gain methods of variation

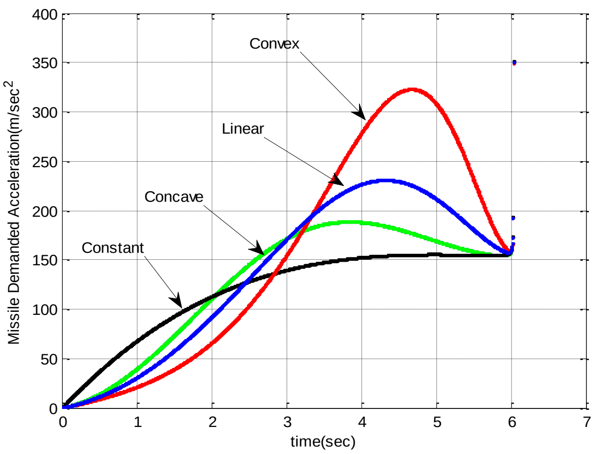

Fig. 7 Missile demanded acceleration using against manoeuvring target

V. PN WITH TVNG IN THE PRESENCE OF NOISE

In PN the commanded acceleration is applied normal to the LOS as shown in Fig. 1, this means that the erroneous measurement of the LOS angle and LOS angle rate will contaminate the generated guidance signal (proportional to the LOS rate) Equation 21, and the direction of the guidance command application (depends on LOS rate direction). In the PN guidance system the missile-target LOS angle and angle rate are the critical variable subjected to noise. The noise considered in this work has Gaussian distribution with mean and variance being chosen to reflect the real situations. The type of noise involved is system noise. System noise is generated inside the guidance system, particularly in the receiver part. The main source of noise in radar receivers is thermal noise because electronics in any conductor at temperature other than absolute zero are always in random motion. There are many other sources of noise associated with receivers including environmental background noise, but in practice it is found that if receiver noise is significant it is largely due to thermal noise. The effect of inserting the system noise on the final miss distance will be investigated in next sections under the assumption of the head on attack scenario and the target initiating manoeuvre at t=0, the initial missile target engagement parameters are given in Table1. A MATLAB code was set up for a two-dimensional missile-target engagement simulation using missile target engagement scenario given in Table 1. The simulation will be divided in to two parts, first part will deal with a high grade tracking system with low output noise, and the second part will consider the low grade tracking system with high output noise. The simulation results will be discuss in the following sections.

TABLE I
MISSILE TARGET ENGAGEMENT SCENARIO

	Initial parameters of the missile and target engagement scenario	
	Target	**Missile**
1	3g. target manoeuvre	-20 Deg. heading error
2	Vt = 1000; target speed (m)	vm = 2000; missile speed (m)
3	Rtx = 40000	Rmx = 10000
5	Rty = 10000	Rmy = 10000
6	TVNG = 1 : 5	
7	System Noise = Gauss Distribution	
8	Total Flight Time = 6.035 sec	
9	Head on Attack scenario	

A. *TVNG With Low Noise Tracking system*

The effect of inserting the system noise to the LOS angle rate on the final miss distance, for a missile trajectories guided by PN with TVNG, under the assumption low system noise (high grade tracking system) is shown in Fig. 8. It is clear that the TVNG results in a much tighter trajectory compared with constant gain. Fig. 9 shows the resultants miss distance for the dfferent types of TVNG compared with costant navigation gain.

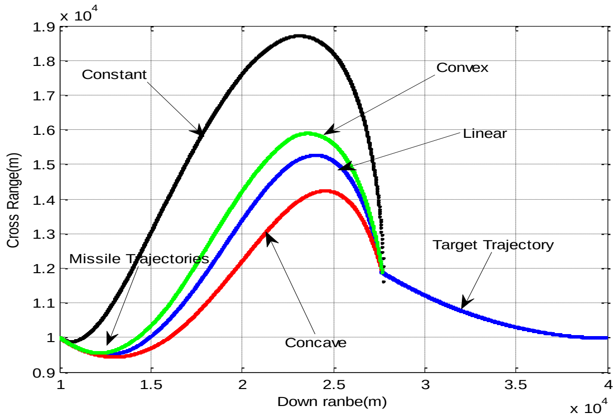

Fig. 8 Missile trajectories using TVNG with low system noise.

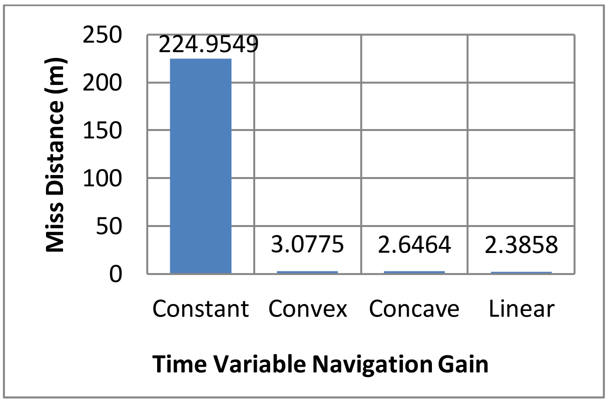

Fig. 9 Miss distances results from TVNG compared with constant navigation gain (low system noise) .

B. TVNG With High Noise Tracking system

High system noise (low grade tracking system) is considered here, the effect of highly noisy measurement of LOS rate signal on the PN with TVNG shown in Fig. 10. It is clear that the TVNG again results in a much tighter trajectory compared with constant gain. Fig. 11 shows the resultants miss distance for the dfferent types of TVNG compared with costant navigation gain. Clearly we can see that miss distance due to high system noise is very large in side constant gain PN compared with that of TVNG. This because PN with a smaller navigation gain is less pronounced to the measurement of the LOS angle rate.

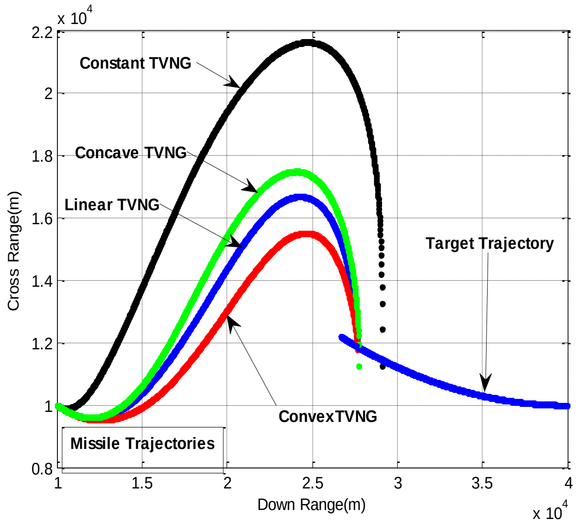

Fig. 10 Missile trajectories using TVNG with high system noise.

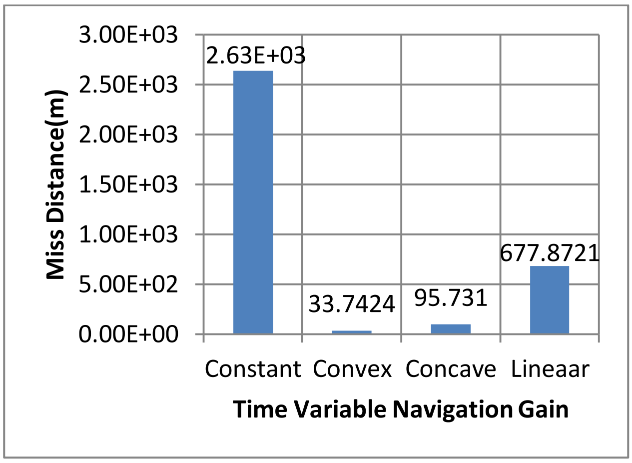

Fig. 11 Miss distances results from TVNG compared with constant navigation gain (high system noise)

VI. Conclusions

In this paper, an investigation study is performed on the PN law of homing guidance system. PN homing guidance law in which the rate of change of the missile heading is proportional to the rate of rotation of the line-of-sight through a constant called navigation gain. The kinematic model investigation showed that PN with a higher navigation gain results in shortest trajectory and minimum control effort. Noise analysis results leads to development of time variable navigation gain (TVNG). In TVNG the navigation gain varies from small gain value(N=1) to a large gain near intercept. This makes use of the advantages of pursuit vulnerability against noise and the guidance accuracy with minimum control effort of the PN. TVNG results in reducing noise effect and hence the final miss distance.

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