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Hybrid Projective Chaos Synchronization by Adaptive Feedback Controllers

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***Abstract*—Adaptive feedback controllers based on Lyapunov stability theory for chaos control and hybrid projective synchronization of the Chen-Lee chaotic system are proposed, in this paper. Firstly, the chaos control to a fixed equilibrium point is presented. Secondly, the hybrid projective synchronization between two identical chaotic systems is developed. Finally, we propose a secure communication scheme based on the adaptive studied projective synchronization property of the Chen-Lee chaotic system. Numerical simulations are demonstrated to verify and illustrate, clearly, the effectiveness of the proposed control strategy.**

Keywords—Lyapunov stability theory; Adaptive feedback controllers; Chaos control; Hybrid projective synchronization; Secure communication.

I. Introduction

Chaotic systems have complex dynamical behaviours that possess some special features, such as excessive sensitivity to initial conditions, broad spectrums of Fourier transform, bounded and fractal properties of the motion in the phase space, and so on. In fact, since Pecora and Carroll [1] showed that it is possible to realize chaos synchronization through a simple coupling, synchronization in coupled chaotic dynamical systems has attracted considerable attention and has been found some potential applications in several fields. Ever since, different types of synchronization phenomena have been observed and investigated in a variety of chaotic systems [2-5].

Recently, hybrid projective synchronization was proposed. It can be considered as an extension of projective synchronization because complete synchronization and anti-synchronization are both its special cases. It is worthy of study because the response signals can be any proportional to the drive signals by adjusting the factors and it can be used to extend binary digital to variety M-nary digital communications for achieving fast communication.

Motivated by this idea, Xu introduced three types of feedback controllers to the drive system to conduct the scaling factor onto a desired value, respectively [6-8]. Indeed, the proposed approach has certain significance for reducing the cost as well as the complexity for transmitting confidential data through using the chaotic masking strategy [7].

The organization of this paper is as follows. In Section 2, the Chen-Lee chaotic system is simply introduced and a new adaptive feedback controller is developed for controlling it to a desired equilibrium point. Then, in Section 3, an adaptive scheme for hybrid projective synchronization is proposed and numerical illustrations are given to prove the occurrence of the hybrid projective synchronization in the Chen-Lee chaotic system. Finally, a scheme of secure communication based on the adaptive hybrid projective synchronization of the Chen-Lee chaotic system is presented, in Section 4.

II. Chaos Control of the Chen-Lee Chaotic System

A. System Description

Chen-Lee system [9] is a new 3D chaotic system which was proposed by Chen and Lee. It takes, in the state space, the description form given as follows:

$$\begin{cases} \dot{x}_1(t) = ax_1(t) - x_2(t)x_3(t) \\ \dot{x}_2(t) = -bx_2(t) + x_1(t)x_3(t) \\ \dot{x}_3(t) = -cx_3(t) + \dfrac{1}{3}x_1(t)x_2(t) \end{cases} \tag{1}$$

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where $x = [x_1 \quad x_2 \quad x_3]^T$ is the vector of state variables, a, b, c are positive constant parameters, and $0 < a < b + c$ to allow the system to generate chaos.

The system (1) is robust to various small perturbations due to its highly symmetric structure, and it is dissipative. Its chaotic attractor is shown, in Fig. 1., for $a = 5$, $b = 10$ and $c = 3.8$.

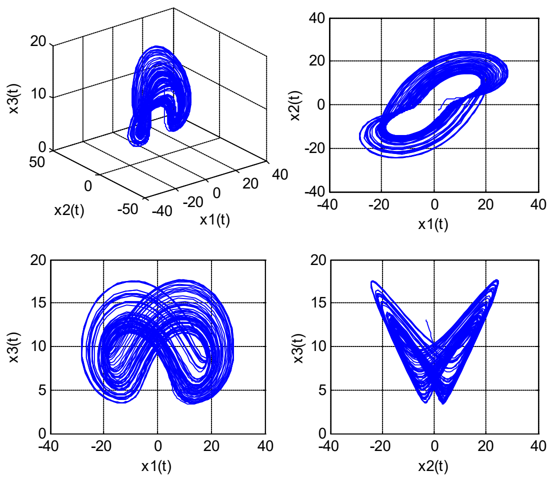


Fig. 1. Various views of the Chen-Lee chaotic attractor

B. Proposed Approach Controlling Chaos of the Chen-Lee System

In this subsection, chaotic system (1) will be controlled to its unstable equilibrium point $O(0,0,0)$ via an adaptive linear feedback controller which only includes one state variable. The controller can be designed as follows:

$$u_i(t) = -kx_i(t),\ u_j(t) = 0\ \forall\ \underset{i \neq j}{i, j} = 1,2,3 \tag{2}$$

where the feedback parameter gain k is adapted according to the following update law:

$$\begin{aligned} &\dot{k}(t) = k_1 x_i^2(t)\ \forall\ i = 1,2,3 \\ &k(0) = 0,\ k_1 > 0 \end{aligned} \tag{3}$$

According to (2) and (3), the controller associated with adaptive update law can be chosen by respect to the above form:

$$\begin{aligned} &u_1(t) = -kx_1(\mathrm{t}),\ u_2(t) = u_3(t) = 0 \\ &\dot{k}(t) = k_1 x_1^2(t),\ k(0) = 0,\ k_1 > 0 \end{aligned} \tag{4}$$

and the controlled chaotic system is, then, considered as:

$$\begin{cases} \dot{x}_1(t) = ax_1(t) - x_2(t)x_3(t) + u_1(t) \\ \dot{x}_2(t) = -bx_2(t) + x_1(t)x_3(t) \\ \dot{x}_3(t) = -cx_3(t) + \dfrac{1}{3}x_1(t)x_2(t) \end{cases} \tag{5}$$

Hence, it comes the following theorem for stabilizing the origin $O(0,0,0)$ of the considered chaotic system (5).

Theorem. *The controlled chaotic system (5) will globally and asymptotically converge to the unstable equilibrium point $O(0,0,0)$ under the controller of form (2) with the update law (4).*

Proof. By introducing a positive and definite candidate Lyapunov function defined by:

$$\begin{aligned} V_1(t) &= \frac{1}{2}\left(\alpha_1 x_1^2(t) + \beta_1 x_2^2(t) + \gamma_1 x_3^2(t)\right) \\ &\quad + \frac{1}{2k_1}k(t)^2 \end{aligned} \tag{6}$$

so, its corresponding derivative function will be expressed as follows:

$$\begin{aligned} \dot{V}_1(t) &= \alpha_1 x_1(t)\dot{x}_1(t) + \beta_1 x_2(t)\dot{x}_2(t) \\ &\quad + \gamma_1 x_3(t)\dot{x}_3(t) + \frac{1}{k_1}k(t)\dot{k}(t) \end{aligned} \tag{7}$$

which yields to:

$$\begin{aligned} \dot{V}_1(t) &= -\left(-\alpha_1 a + \alpha_1 k - k\right)x_1^2(t) \\ &\quad - \beta_1 b x_2^2(t) - \gamma_1 c x_3^2(t) \\ &\quad - \left(\alpha_1 - \beta_1 - \frac{1}{3}\gamma_1\right)x_1(t)x_2(t)x_3(t) \end{aligned} \tag{8}$$

Thus, to simplify the expression of the derivative function $\dot{V}_1(t)$, let's adjust adequately the parameters α_1, β_1 and γ_1 so that:

$$\alpha_1 - \beta_1 - \frac{1}{3}\gamma_1 = 0 \tag{9}$$

Among several possibilities, let's consider the following one:

$$\begin{cases} \alpha_1 = 2 \\ \beta_1 = 1 \\ \gamma_1 = 3 \end{cases} \tag{10}$$

Then, (8) takes the new form (11):

$$\dot{V}_1(t) = -\left(-2a + k\right)x_1^2(t) - bx_2^2(t) \\ -3cx_3^2(t) \tag{11}$$

which can be written as follows:

$$\dot{V}_1(t) = -x^T(t)P_1x(t) \tag{12}$$

with:

$$P_1 = \begin{bmatrix} -2a+k & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 3c \end{bmatrix} \tag{13}$$

Consequently, by choosing the feedback gain k such that:

$$k > 2a \tag{14}$$

so, it is obvious that the symmetric matrix P_1 is positive definite and $\dot{V}_1(t)$ is negative semi-definite since $a > 0$, $b > 0$ and $c > 0$.

According to:

$$\int_0^t \lambda_{\min}\left(P_1\right)\left\|x(t)\right\|^2 dt \le \int_0^t x^T(t)P_1x(t)dt \\ = -\int_0^t \dot{V}_1(t)dt \le V_1(0) \tag{15}$$

where $\lambda_{\min}\left(P_1\right)$ is the smallest eigenvalue of P_1, $x(t)$ and $\dot{x}(t)$ are both bounded. It follows that $\dot{V}_1(t)$ is uniformly continuous. Based on Barbalat's lemma, $\dot{V}_1(t) \to 0$ as $t \to +\infty$. Subsequently, the system (5) converges to $O\left(0,0,0\right)$ within a shorter time under the suitable designed controller with respect to the update law (4).

This completes the proof.

Numerical simulations demonstrate the performance of the proposed method for conducting the considered chaotic system to one chosen equilibrium point $O\left(0,0,0\right)$. The initial conditions of the state vector are $x_1(0) = 1$, $x_2(0) = 2$ and $x_3(0) = 3$, the initial condition of the adaptive feedback gain is $k(0) = 0$, and the constant coefficient k_1 is set to be 21.

At the outset, Fig. 2. shows the time responses of the state variables x_1, x_2 and x_3 when controller is switched off, relatively to the chaotic system (1). Then, Fig. 3. illustrates, clearly, the asymptotic convergence of the three above-mentioned state variables for the controlled chaotic system (5). It is observed that chaotic behaviour is suppressed by means of the single proposed scalar controller $u_1(t) = -kx_1(t)$, such that $\dot{k}(t) = k_1x_1^2(t)$.

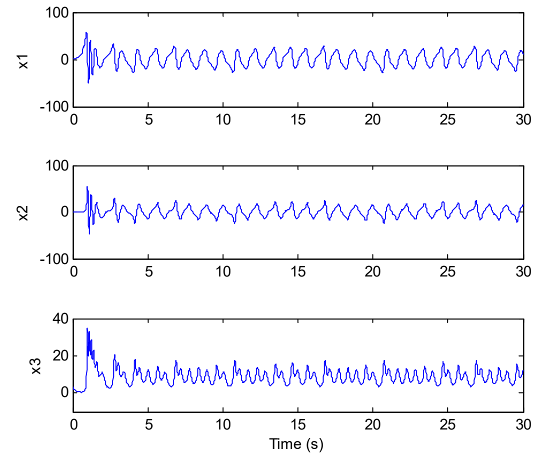


Fig. 2. Dynamics of the state variables of the Chen-Lee chaotic system when the adaptive controller is deactivated

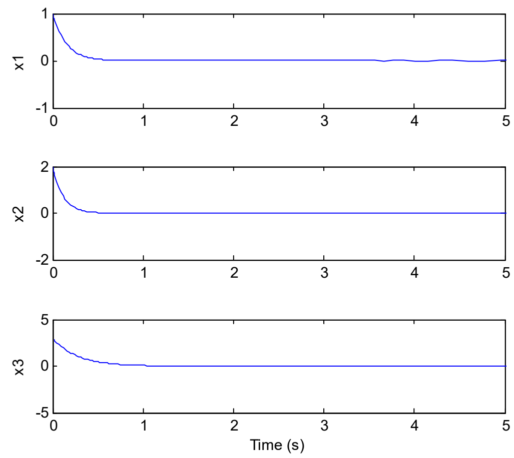


Fig. 3. State trajectories of the Chen-Lee controlled chaotic system

It is relevant to denote that the proposed single scalar adaptive control strategy (2) with the adaptive update law (3) can be applied to a class of general 3D chaotic system expressed by:

$$\dot{x}(t) = Ax(t) + G(x) \tag{16}$$

where $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \in \mathbb{R}^3$ is the state vector, $A \in \mathbb{R}^{3\times3}$ is a constant matrix and $g(x) = \begin{bmatrix} g_1(x) & g_2(x) & g_3(x) \end{bmatrix}^T$ is a nonlinear vector, satisfying:

$$\begin{aligned} &a_1 x_1(t) g_1(x) + a_2 x_2(t) g_2(x) \\ &+ a_3 x_3(t) g_3(x) = 0 \end{aligned} \tag{17}$$

where a_1, a_2 and a_3 are three constants.

III. Hybrid Projective Synchronization for Chaotic Systems

The main aim of this part is to study the projective synchronization in coupled chaotic systems, by adopting the adaptive feedback idea proposed in the previous section.

A. Hybrid Projective Chaos Synchronization by Adaptive Feedback Control Law

Considering two dynamical drive and response chaotic systems described, respectively, by:

$$\dot{x}(t) = f(x) \tag{18}$$

and:

$$\dot{y}(t) = g(y) + u(x, y) \tag{19}$$

where $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$ and $y = \begin{bmatrix} y_1 & y_2 & \dots & y_n \end{bmatrix}^T$ are the state variables of the drive system (18) and the response system (19), respectively, $f(.)$ and $g(.)$ are $n \times 1$ nonlinear vectorial functions, and $u(x, y) = \begin{bmatrix} u_1(x, y) & \dots & u_n(x, y) \end{bmatrix}^T$ is the nonlinear control vector to be determined.

If there exists a nonzero constant matrix $h = diag\begin{bmatrix} h_1 & h_2 & \dots & h_n \end{bmatrix}$ such that:

$$\lim_{t\to+\infty} \left| y_i(t) - h_i x_i(t) \right| = 0 \ \forall \ i = 1, 2, \dots, n \tag{20}$$

then the response system and the drive system are said to be in hybrid projective synchronization. In particular, the coupled drive-response system achieves complete synchronization when all values of coefficients h_i are equal to (1) and the two considered chaotic systems are said to be in anti-synchronization when all values of coefficients h_i are equal to (-1).

B. Case of Two Coupled Chen-Lee Chaotic Systems

In this stage, we study the hybrid projective synchronization of two identical Chen-Lee chaotic systems. The response system corresponding to the drive system (1) is defined as follows:

$$\begin{cases} \dot{y}_1(t) = ay_1(t) - y_2(t)y_3(t) + u_1(t) \\ \dot{y}_2(t) = -by_2(t) + y_1(t)y_3(t) + u_2(t) \\ \dot{y}_3(t) = -cy_3(t) + \dfrac{1}{3} y_1(t)y_2(t) + u_3(t) \end{cases} \tag{21}$$

where $u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$ is the nonlinear control vector. System (1) and system (21) are in hybrid projective synchronization as long as:

$$\lim_{t\to+\infty} \left| y_i(t) - h_i x_i(t) \right| = 0 \ \forall \ i = 1, 2, 3 \tag{22}$$

Let's define the state error vector as $e(t) = y(t) - hx(t),\ e = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^T$, namely:

$$\begin{cases} e_1(t) = y_1(t) - h_1 x_1(t) \\ e_2(t) = y_2(t) - h_2 x_2(t) \\ e_3(t) = y_3(t) - h_3 x_3(t) \end{cases} \tag{23}$$

where $h = diag\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}$ and h_1, h_2, h_3 are different desired scaling factors for hybrid projective synchronization. In such a way, the dynamical error system between the response system (21) and the drive system (1) can be written as follows:

$$\begin{cases} \dot{e}_1(t) = ae_1 - e_2e_3 - h_2x_2e_3 - h_3x_3e_2 \\ \qquad -\left(h_2h_3 - h_1\right)x_2x_3 + u_1 \\ \dot{e}_2(t) = -be_2 + e_1e_3 + h_1x_1e_3 + h_3x_3e_1 \\ \qquad +\left(h_1h_3 - h_2\right)x_1x_3 + u_2 \\ \dot{e}_3(t) = -ce_3 + \dfrac{e_1e_2}{3} + \dfrac{h_1x_1e_2}{3} + \dfrac{h_2x_2e_1}{3} \\ \qquad + \dfrac{\left(h_1h_2 - h_3\right)x_1x_2}{3} + u_3 \end{cases} \tag{24}$$

The main goal is to design a controller such that the state errors fulfil:

$$\lim_{t\to+\infty} e_1(t) = \lim_{t\to+\infty} e_2(t) = \lim_{t\to+\infty} e_3(t) = 0 \tag{25}$$

then the global and asymptotical stability of the error dynamical system (24) means that system (1) and system (21) are in hybrid projective synchronization.

Let's consider the control functions u_1, u_2 and u_3 in the following form:

$$\begin{cases} u_1 = \left(h_2 h_3 - h_1\right) x_2 x_3 \\ u_2 = -\left(h_1 h_3 - h_2\right) x_1 x_3 \\ u_3 = -\dfrac{\left(h_1 h_2 - h_3\right) x_1 x_2}{3} \\ \quad -\dfrac{2 h_1 x_1 e_2}{3} - \dfrac{e_1 e_3}{3} - k e_3 \end{cases} \qquad (26)$$

with k the feedback gain adjusted according to the following update law:

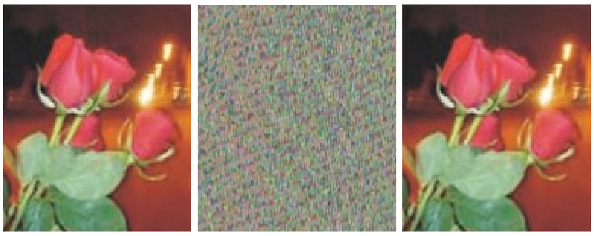
$$\dot{k} = k_2 e_1^2,\ k(0) = 0,\ k_2 > 0 \qquad (27)$$

As a result, for whichever initial conditions, the drive system (1) and the response system (21) are globally and asymptotically hybrid projective synchronized through the proposed nonlinear feedback controllers (26), by respect to the update law (27).

IV. Application for Color Image Encryption and Decryption

In this part, the problem of hybrid projective synchronization between two identical chaotic Chen-Lee systems is applied to a new chaos-based image cryptosystem, in order to illustrate the feasibility of the theoretical proposed approach. The input of the considered cryptosystem is the plain image which will be encrypted.

Primarily, we form a vector with three layers in the RGB format containing the image colors. After that, the chaotic signal of the drive transmitter system is added to the image, to further enhance the complexity of the considered cryptosystem and, in this manner, improving the security of the image transmission process. Afterward, the image is successfully recovered through the subtraction between the encrypted image and the response receiver chaotic signal. At last, the three layers are joined in order to form the color image, as presented in Fig. 4.



(a) (b) (c)

Fig. 4. (a) Original image, (b) Encoded image, (c) Decoded image

V. Conclusion

In this paper, the confident communication problem based on the hybrid projective synchronization of chaotic systems is studied. The asymptotic convergence of the errors between the states of the drive system and the states of the response system is proven by means of Lyapunov stability theory. The emitted image is modulated into the parameter information of the transmitter system and the corresponding receiver is designed so that it is able to retrieve, secretly, the former image.

By referring to simulation results, it can be decided that the developed theoretical approach is achievable and resourceful, from the time when it is successfully exploited to privately transmit and recover one chosen color image.

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