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Consensus of Double-Integrator Multi-Agent Systems Using Decentralized Model Predictive Control

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***Abstract*—This paper studies the consensus for a group of agents which have a discrete-time double-integrator dynamics. To solve this problem, a Model Predictive Control (MPC) scheme is introduced. The proposed control protocol is decentralized and is designed by combining graph theory with a predictive control algorithm to take into account the switches on the communication topology. The predictive strategy is used to estimate input and output of agent through a receding horizon. Contrary to many existing works, the cost function is designed using the difference between two consecutive inputs. The controller has integrator properties to eliminate steady-state errors. Finally, some simulation results are given to show the effectiveness of the proposed model predictive control.**

I. Introduction

In recent years, many works on multi-agent systems (MAS) have been done in many areas, e.g. formation control [1] [3], target tracking [4] [5], optimal coverage [6], distributed monitoring [7] [8], etc. Compare with a single agent, multiple agents may perform a mission more efficiently and provide higher flexibility during the task execution. One of fundamental problems on MAS is to design decentralized control protocols to guarantee agreement from all agents regarding a certain quantity of interest via local interaction, called consensus [9] [10]. Some results on consensus schemes can be categorized into two types depending on whether there is a leader or not. For instance, [11] [12] investigated when there is no leader or when the leader is static.

Compared with the continuous-time systems, discrete-time systems are more suitable for practical applications. Some interesting works related to the topic of first-order discrete-time consensus stability were reported in [13] [14] [15]. The main objective of [13] was to theoretically study the coordination of a group of autonomous agents using Vicsek model. In [14], some consensus protocols for discrete-time systems with switching topology were provided and the robustness against time delays was analyzed. These two kinds of protocols are based using the same data at two time-steps. The dynamic behavior of discrete-time multi-agent systems with general communication topologies was considered in [15]. For topologies that have a spanning tree, the consensus problem was studied. It was proved that the states of internal agents converge to a convex combination of boundary agents in the case of communication time delays.

Model Predictive Control (MPC) is a form of control in which the output of the system can be predicted from some prediction horizon. The output of the MPC controller is determined based on input and output at a previous time and the control signal along control horizon. Futhermore, MPC has ability to handle control and state constraints for discrete-time systems [16]. This method can be applied for the control of a group of agents by letting each agent solve, at each step, a constrained finite-time optimal control problem involving the state of neighboring agents. For agents modeled by a discrete-time system, [17] proposed decentralized MPC schemes with control input constraints and showed that under the proposed decentralized schemes, multi-agent systems with single- and double-integrator dynamics asymptotically achieves consensus under mild assumptions. However, it was assumed that the control horizon equals the prediction control, which reduced the degree of freedom for the controller design. To remove the problem in degree of freedom in controller design, [19] proposed a consensus scheme for discrete-time single-integrator MAS under switching directed interaction graphs where the control horizon can be arbitrarily picked from one to prediction horizon. Another result of MPC is [18] which proposed a MPC strategy to increase the consensus convergence rate in MASs under some special communication networks.

This paper deals with the consensus problem for discrete-time double-integrator MAS. The objective is to design a MPC protocol by combining graph theory with a predictive control algorithm such that the states of all agents reach an

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agreement while taking into account the switches on the communication topology. Since our focus is to minimize tracking errors between agents, we try to design the criteria function using the difference between two consecutive inputs. Therefore, the controller has integrator properties to eliminate steady-state errors.

The paper is organized as follows. In Section 2, some concepts on algebraic graph theory are given and the consensus problem is formulated. In Section 3, the controller design which solves the consensus problem is discussed for discrete-time double-integrator systems. In Section 4, some simulations results are given to show the effectiveness of the proposed controller. Finally, concluding remarks are drawn in Section 5.

Notations: The definitions $\mathbb{R}^m$, $\mathbb{R}^{mn}$ and $\mathbb{R}^{nn}$ denote the sets of m-dimensional real column vectors, mn-dimensional real column vectors and $n\times n$ dimensional real matrices, respectively. $\|.\|$ indicates the Euclidean norm.The definition $*(k+l|k)$ denotes the prediction value $*$ at instant $k+l$ based on the currently available information at instant k.

II. Recalls on graph theory

Consider a MAS composed of n agents. In this paper, at time k, the communication network among agents is illustrated by a digraph (directed graph) denoted as $\mathscr{G}(k)=\{\mathscr{V},\mathscr{E}(k)\}$. $\mathscr{V}=\{1,\dots,N\}$ is the set of nodes which correspond to agents. $\mathscr{E}(k)\subseteq\{\mathscr{V}\times\mathscr{V}\}$ is the set of edges. An edge $(j,i)\in\mathscr{E}(k)$, with $i\neq j$, exists if at time k agent i can receive information from agent j. At time k, the set of neighbors to the node $i\in\mathscr{V}$ is $\mathscr{N}_i(k)=j\in\mathscr{V}:(j,i)\in\mathscr{E}(k)$ and $|\mathscr{N}_i(k)|, i\in\mathscr{V}$, is the valency or degree of i-th node at time k. $\mathscr{A}(k)=(a_{ij}(k))\in\mathbb{R}^{N\times N}$ is defined as the weighted adjacency matrix of $\mathscr{G}(k)$ where $a_{ij}(k)>0$ if $(j,i)\in\mathscr{E}(k)$ and $a_{ij}(k)=0$, otherwise. The graph Laplacian matrix of $\mathscr{G}(k)$ is defined as $\mathscr{L}(k)=(l_{ij}(k))\in\mathbb{R}^{N\times N}$ with $l_{ii}(k)=\sum_{j=1,\,j\neq i}^{N}a_{ij}(k)$ and $l_{ij}(k)=-a_{ij}(k)$ for $i\neq j$. A digraph $\mathscr{G}(k)$ contains a spanning tree if there is a node (called root node) such that there is a directed path from this node to every other node. It is connected if there is a path between any two nodes.

In this paper, it is assumed that the digraph is time-varying. Such time-varying graphs can be found in many engineering applications due to creation and failure of communication links, reconfiguration of formations, presence of obstacles and so on. The corresponding Laplacian is $L(k)=L_{\sigma(k)}$ where $\sigma:\{0,\dots\}\to\mathscr{Q}$ is a switching signal and $\mathscr{Q}=\{1,\cdots,M\}$ is a finite index set. The changes of σ are the switching times s_q, $q=0,1,\cdots$ with $s_0=t_0$.

In the following, the time-varying graph is supposed to be jointly connected. It means that the union of graphs $\mathscr{G}(k)$ whose edge set is the union of edge sets of all graphs $\mathscr{G}(k)$ is connected.

A. Problem Statement

Let us consider the MAS consisting of n agents with discrete-time linear dynamics, described by:

$$\mathbf{x_i}(k+1)=\mathbf{A}\mathbf{x_i}(k)+\mathbf{B}u_i(k),\qquad i=1,2,...,n \tag{1}$$

with $\mathbf{A}=\begin{bmatrix}1&\delta\\0&1\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}\frac{1}{2}\delta^2\\\delta\end{bmatrix}$.

Here $\mathbf{x_i}\in\mathbb{R}^2$ (resp. $u_i(k)\in\mathbb{R}$) indicates the state (resp. control input) of the ith agent at time k. $\delta\in\mathbb{R}^+$ is the sampling period.

In this paper, the control objective is to design a decentralized protocol such that the MAS achieves consensus. It means that

$$\lim_{k\to\infty}\|\mathbf{x}_i(k)-\mathbf{x}_j(k)\|=0\qquad\forall i,j\in\mathscr{V},i\neq j \tag{2}$$

for any initial state $x_i(0)$.

III. Decentralized model predictive scheme

In this section, we propose to solve the problem of MAS consensus using a model predictive scheme.

Let $H_p\geq H_u\geq 1$ denote the length of the prediction horizon (resp. length of the control horizon). Contrary to many existing works, we use the difference Δu_i between two consecutive inputs to eliminate the steady state errors, i.e.

$$\Delta u_i(k+l)=\begin{cases}u_i(k+l)-u_i(k+l-1) & 0\leq l\leq H_u-1\\ 0 & H_u\leq l\leq H_p\end{cases} \tag{3}$$

Using the difference between two consecutive inputs, one can derive the following predictions at time k

$$\begin{aligned}\mathbf{x_i}(k+1|k)&=\mathbf{A}\mathbf{x_i}(k)+\mathbf{B}u_i(k)\\&=\mathbf{A}\mathbf{x_i}(k)+\mathbf{B}(\Delta u_i(k)+u_i(k-1))\end{aligned}$$

$$\begin{aligned}\mathbf{x_i}(k+2|k)&=\mathbf{A}\mathbf{x_i}(k+1|k)+\mathbf{B}u_i(k+1)\\&=\mathbf{A}\big(\mathbf{A}\mathbf{x_i}(k)+\mathbf{B}(\Delta u_i(k)+u_i(k-1))\big)\\&\quad+\mathbf{B}(\Delta u_i(k+1)+\Delta u_i(k)+u_i(k-1))\\&=\mathbf{A}^2\mathbf{x_i}(k)+(\mathbf{AB}+\mathbf{B})\Delta u_i(k)\\&\quad+\mathbf{B}\Delta u_i(k+1)+(\mathbf{AB}+\mathbf{B})u_i(k-1)\end{aligned}$$

$$\begin{aligned}\mathbf{x_i}(k+3|k)&=\mathbf{A}\mathbf{x_i}(k+2|k)+\mathbf{B}u_i(k+2)\\&=\mathbf{A}^3\mathbf{x_i}(k)+(\mathbf{AB}^2+\mathbf{AB}+\mathbf{B})\Delta u_i(k)\\&\quad+(\mathbf{AB}+\mathbf{B})\Delta u_i(k+1)\\&\quad+\mathbf{B}\Delta u_i(k+2)\\&\quad+(\mathbf{A}^2\mathbf{B}+\mathbf{AB}+\mathbf{B})u_i(k-1)\end{aligned}$$

$$
\begin{aligned}
\mathbf{x_i}(k+H_p|k) \;=\; & \mathbf{A}^{H_p}\mathbf{x_i}(k)+\left(\sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B}\right)\Delta u_i(k) \\
& +\left(\sum_{i=0}^{H_p-2}\mathbf{A}^i\mathbf{B}\right)\Delta u_i(k+1)+\dots \\
& +\mathbf{B}\Delta u_i(k+H_p-1) \\
& +\left(\sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B}\right)u_i(k-1)
\end{aligned}
$$

Hence, one can write the previous equalities as follows:

$$
\begin{bmatrix}\mathbf{x_i}(k+1|k)\\ \mathbf{x_i}(k+2|k)\\ \mathbf{x_i}(k+3|k)\\ \vdots \\ \mathbf{x_i}(k+H_p|k)\end{bmatrix} = \begin{bmatrix}\mathbf{A}\\ \mathbf{A}^2\\ \mathbf{A}^3\\ \vdots\\ \mathbf{A}^{H_p}\end{bmatrix}\mathbf{x_i}(k)+\begin{bmatrix}\mathbf{B}\\ \mathbf{AB}+\mathbf{B}\\ \mathbf{A}^2\mathbf{B}+\mathbf{AB}+\mathbf{B}\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B}\end{bmatrix}u_i(k-1)
$$

$$
+\begin{bmatrix}\mathbf{B} & 0 & 0 & 0\\ \mathbf{AB}+\mathbf{B} & \mathbf{B} & 0 & 0\\ \mathbf{A}^2\mathbf{B}+\mathbf{AB}+\mathbf{B} & \mathbf{AB}+\mathbf{B} & \mathbf{B} & \vdots\\ \vdots & \vdots & \ddots & \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B} & \sum_{i=0}^{H_p-2}\mathbf{A}^i\mathbf{B} & \dots & \mathbf{B}\end{bmatrix}\begin{bmatrix}\Delta u_i(k)\\ \Delta u_i(k+1)\\ \Delta u_i(k+2)\\ \vdots\\ \Delta u_i(k+H_p-1)\end{bmatrix}
$$

In a compact form, the dynamic system becomes as follows:

$$
\mathbf{X_i}(k)=\mathbf{P_x}\mathbf{x_i}(k)+\mathbf{P_u}u_i(k-1)+\mathbf{P}_\triangle\Delta U_i(k) \tag{4}
$$

with $\mathbf{X_i}(k)=\begin{bmatrix}\mathbf{x_i}(k+1|k)\\ \mathbf{x_i}(k+2|k)\\ \mathbf{x_i}(k+3|k)\\ \vdots\\ \mathbf{x_i}(k+H_p|k)\end{bmatrix}$, $\Delta U_i(k)=\begin{bmatrix}\Delta u_i(k)\\ \Delta u_i(k+1)\\ \Delta u_i(k+2)\\ \vdots\\ \Delta u_i(k+H_p-1)\end{bmatrix}$,

$$
\mathbf{P_x}=\begin{bmatrix}\mathbf{A}\\ \mathbf{A}^2\\ \mathbf{A}^3\\ \vdots\\ \mathbf{A}^{H_p}\end{bmatrix},\ \mathbf{P_u}=\begin{bmatrix}\mathbf{B}\\ \mathbf{AB}+\mathbf{B}\\ \mathbf{A}^2\mathbf{B}+\mathbf{AB}+\mathbf{B}\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B}\end{bmatrix} \text{ and}
$$

$$
\mathbf{P}_\triangle=\begin{bmatrix}\mathbf{B} & 0 & 0 & 0\\ \mathbf{AB}+\mathbf{B} & \mathbf{B} & 0 & 0\\ \mathbf{A}^2\mathbf{B}+\mathbf{AB}+\mathbf{B} & \mathbf{AB}+\mathbf{B} & \mathbf{B} & \vdots\\ \vdots & \vdots & \ddots & \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}^i\mathbf{B} & \sum_{i=0}^{H_p-2}\mathbf{A}^i\mathbf{B} & \dots & \mathbf{B}\end{bmatrix}.
$$

Then, we select the MPC cost function for agent i as follows:

$$
J_i(k)=\|\mathbf{P_x}\mathbf{x_i}(k)+\mathbf{P_u}u_i(k-1)-\mathbf{r_i}(\mathbf{k})\|^2_{\mathbf{Q}}+\|\Delta U_i(k)\|^2_{\mathbf{R}} \tag{5}
$$

where $\mathbf{Q}$ and $\mathbf{R}$ represents the associated state-weighted matrix with and control-weighted matrix with appropriate dimensions, respectively. $\mathbf{r_i}(\mathbf{k})$ denotes the reference state for agent i over the future H_p prediction step and will be defined hereafter.

***Remark** 1:* It should be notify that the proposed cost function (5) is decentralized if the reference state $\mathbf{r_i}(\mathbf{k})$ only depends on the neighboring agents of agent i at time k.

To solve the consensus problem, we want to minimize the tracking errors between agents using the cost function (5), derived from the difference Δu_i between two consecutive inputs. Therefore, let us define the quadratic optimization problem for each agent i as

$$
min\, J_i(k)=\sum_{l=1}^{H_p}\|\mathbf{x_i}(k+l|k)-\mathbf{r_i}(\mathbf{k})\|^2_{\mathbf{Q}}+\sum_{l=0}^{H_u-1}\|\Delta u_i(k+l|k)\|^2_{\mathbf{R}} \tag{6}
$$

subject to:

$$
\mathbf{x_i}(k+l|k)=\mathbf{A}\mathbf{x_i}(k)+\mathbf{B}(\Delta u_i(k)+u_i(k-1)) \tag{7}
$$

The reference state is chosen as

$$
\mathbf{r_i}(k)=\frac{1}{|\mathcal{N}_i(k)|+1}\sum_{j\in\mathcal{N}_i(k)\bigcup i}x_j(k) \tag{8}
$$

It only depends on the states of the ith agent and its neighbors which implies that the cost function (6) is decentralized. Furthermore, it evolves during time according to the communication topology through the set $\mathcal{N}_i(k)$.

IV. Simulation Results

In this section, some simulation results are provided to verify the theoretical analysis. Consider the topology of MAS with $n=4$ agents as shown in Fig. 1 taken from [19]. In the following, the sampling period is set to $\delta=0.1s$.

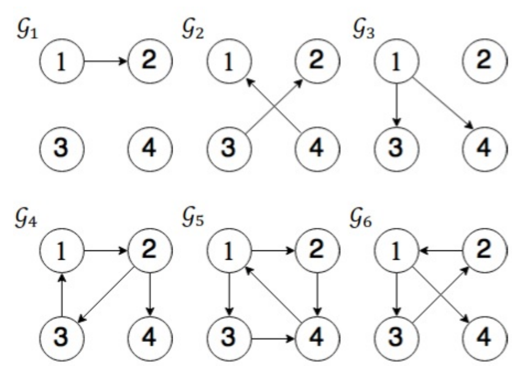

Fig. 1: Switching communication topologies between agents

Let us consider example given in [20]. In this example, the multi-agent system is modeled as, $\forall i=1,\dots,4$,

$$
\mathbf{z_i}(k+1)=\begin{pmatrix}0.6005 & -0.1 & 0.4005\\ -0.1 & 0.1 & 0.1\\ -0.5995 & -1.9 & 1.6005\end{pmatrix}\mathbf{z_i}(k)+\begin{pmatrix}3 & 0\\ 0 & 2\\ 1 & 1\end{pmatrix}\mathbf{v}_i(k) \tag{9}
$$

The initial states of each agents are given as

$$\mathbf{x_1}(0) = \begin{bmatrix}10\\1\\3\end{bmatrix}, \mathbf{x_2}(0) = \begin{bmatrix}0\\1\\3\end{bmatrix}, \mathbf{x_3}(0) = \begin{bmatrix}-2\\3\\0\end{bmatrix}, \mathbf{x_4}(0) = \begin{bmatrix}6\\0\\-2\end{bmatrix}$$

Through an appropriate change of coordinate, model (9) can be easily rewritten as:

$$\mathbf{x_i}(k+1) = \begin{pmatrix}0.3 & 0 & 0\\ 0 & 1 & 0.1\\ 0 & 0 & 1\end{pmatrix}\mathbf{x_i}(k) + \begin{pmatrix}1 & 0\\ 0 & \frac{1}{2}0.1^2\\ 0 & 0.1\end{pmatrix}\mathbf{v}_i(k) \quad (10)$$

It means that the system can be divided into a simple and a double integrator subsystems. Hence, one can easily apply the proposed model predictive controller described in the previous section. The control parameter are selected as $H_p = 3$ and $H_u = 2$.

The consensus problem described in equation (2) is archived using the proposed model predictive control. We can see that the state of all agents converge to a consensus point. Figure 2 shows that the state x_1 achieves the consensus with a settling time less than $2s$. For the state x_2 and x_3, figures 3-4, illustrate that the consensus is archived with settling time less than $3s$.

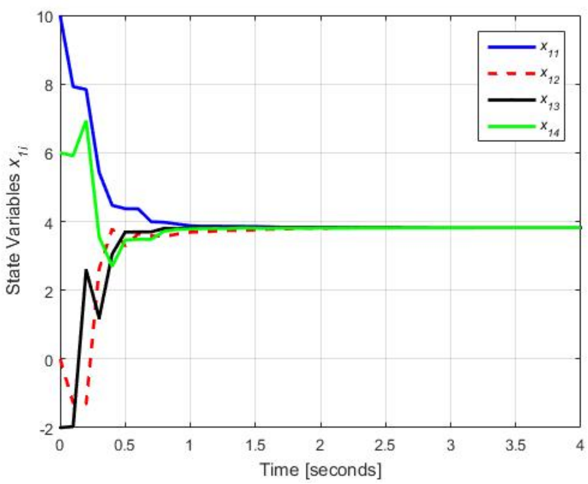

Fig. 2: Evolution of the agent state x_1

The corresponding the control input is given in fig. 5,

Figure 6 and 7 depict the influence of the prediction horizon H_p especially in terms of settling time.

V. Conclusion

In this paper, a model predictive control protocol is developed for consensus of MASs with discrete-time double-integrator dynamics under time-varying directed interaction graphs topologies. The control protocol is decentralized and is designed by combining graph theory with a predictive control algorithm to take into account the switches on the communication topology. The cost function is design using the difference between two consecutive inputs. The controller has integrator properties to eliminate steady-state errors. The settling time of consensus depends on the prediction horizon parameters. Some simulation results have been given to show the effectiveness of the proposed controller.

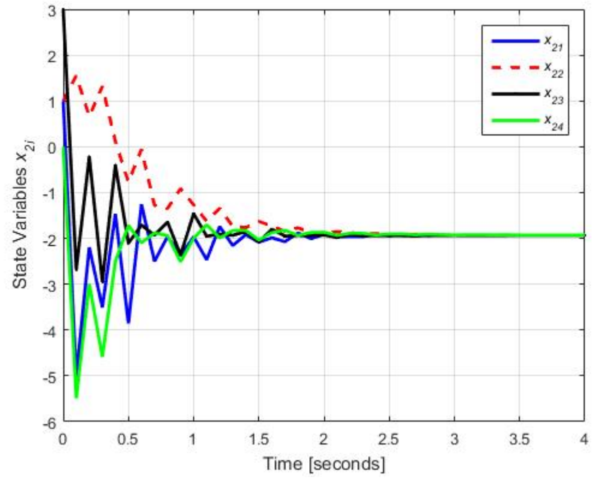

Fig. 3: Evolution of the agent state x_2

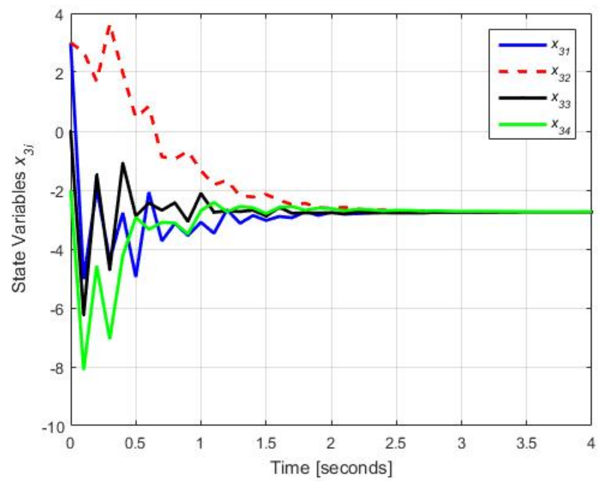

Fig. 4: Evolution of the agent state x_3

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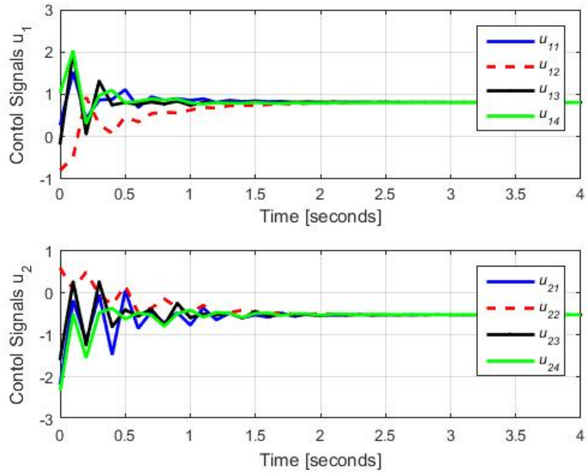

Fig. 5: Evolution of the control input u_1 and u_2

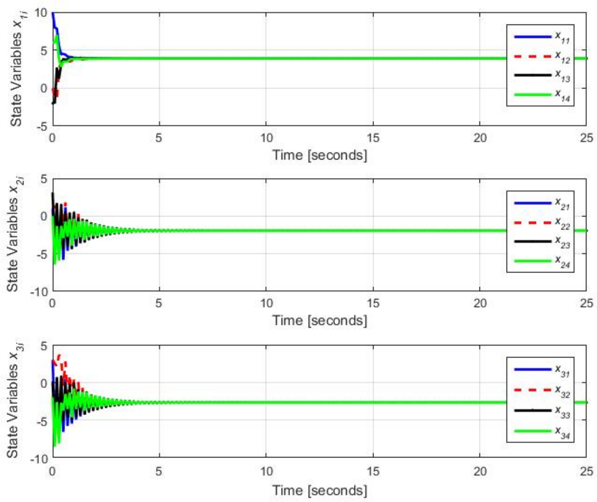

Fig. 6: Evolution of the agent states for $H_p = 4$

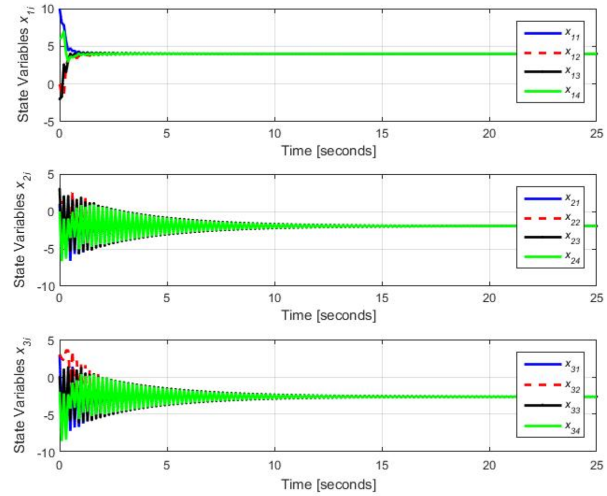

Fig. 7: Evolution of the agent states for $H_p = 5$

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