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Adaptive Fuzzy Control for a Discrete-Time Nonlinear Systems

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Abstract— **This paper, deals with an indirect adaptive fuzzy control of discrete-time MIMO nonlinear systems with parametric variations. The synthesis of the state feedback control law is based on the T-S fuzzy models developed by a local description of the considered system. In the first step, gradient method is used to adjust model parameters locally estimated by the fuzzy model. The state feedback control law based on pole placement is applied to the nonlinear system in the second step. Based on the Lyapunov stability theory, the asymptotic stability of the proposed state feedback adaptive fuzzy control method is studied. Two links robot manipulator arm is used to illustrate the performance of the proposed controller.**

***Keywords*— Adaptive control, T-S fuzzy system, nonlinear systems, discrete-time model, stability analysis**

I. Introduction

The control of nonlinear systems has been the subject of many research works [4]. Fuzzy systems have been successfully applied to many control problems because they do not need an accurate mathematical model of the system under control [12]. In fact, fuzzy control algorithms are introduced based on the T-S fuzzy model [1,2,7,8]. The basic idea of this method is to represent the complex nonlinear system by linear local models. Then, for each one a state feedback control law is calculated. Thereafter, the global control law can be obtained either by combining all local control laws [2]. This design, called Parallel Distributed Compensation PDC, is intuitive and simple.

In the literature of fuzzy adaptive control, many authors are interested in affine continuous systems. They integrate other techniques like the sliding mode control to develop an update parameters law such as [13]. Some others are based on the recursive last square method to propose update laws [14].

Wang and Tanaka have discussed the design and the stability of discrete single-input single-output fuzzy dynamic control systems.

The main goal of this work, is to propose an indirect adaptive algorithm to design stable fuzzy control law for discretes non affines nonlinear systems, where a series-parallel T-S fuzzy model representation has been used.

The paper is organized as follows: In Section II, the description of the Takagi Sugeno fuzzy model is formulated. Section III presents the proposed scheme of an indirect adaptive fuzzy control. In section IV, the stability conditions of the proposed method are discussed. Section V, present a numerical example to illustrate the performance of the studied approch. The last section contains conclusions and final remarks.

II. Takagi-Sugeno Fuzzy Model

We consider the discrete-time multi-input multi-output (MIMO) nonlinear systems described by following equations:

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k) = g(x(k)) \end{cases} \quad (1)$$

where f and g are two nonlinear functions.

$x(k) = \left[x_1(k)\ x_2(k) \ldots x_n(k)\right]^T \in \Re^n$, $\mathrm{u}(k) \in \Re^m$

and $y(k) \in \Re^q$ are the measurable state vector, the input vector and the system output vector, respectively at the discrete time k.

The discrete nonlinear system (1) can be represented by a Takagi-Sugeno series-parallel fuzzy dynamic model. It's described by a linear local model for each fuzzy rule [2]. The i^{th} rule of the fuzzy model has the following form:

$$R^i: \text{ if } z_1(k) \text{ is } M_{i1} \text{ and} \ldots \text{and } z_p(k) \text{ is } M_{ip}$$

$$\text{then } \begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ y(k) = C_i x(k) \end{cases} \quad i = 1, 2 \ldots r \quad (2)$$

Here, M_{ip} is the fuzzy set and r is the number of rules, $A_i \in \Re^{n\times n}$ $B_i \in \Re^{n\times m}$ and $C_i \in \Re^{q\times n}$ are the state matrix, the input matrix and the output matrix, respectively. $z_1(k), \ldots z_p(k)$ are known premise variables. The global states and outputs of the fuzzy system are given as follows:

$$x(k+1) = A(k)x(k) + B(k)u(k) \quad (3)$$

where

$$A(k) = \sum_{i=1}^{r} \alpha_i(z(k)) A_i(k)$$

and $B(k) = \sum_{i=1}^{r} \alpha_i(z(k)) B_i(k)$

$$y(k) = C(k)x(k) \quad (4)$$

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where $C(k) = \sum_{i=1}^{r} \alpha_i(z(k))C_i$

$z(k) = \left[z_1(k)\ z_2(k)...z_n(k)\right]$

and $\quad w_i(z(k)) = \prod_{j=1}^{p} \mu_{i,j}(z_j(k)) \quad i = 1, 2...r$

$\mu_{ij}(z_j(k))$ is the appartenance degree of the membership function to the fuzzy set M_{ij}

$$\sum_{i=1}^{r} \alpha_i(z(k)) = \frac{w_i(z(k))}{\sum_{i=1}^{r} w_i(z(k))} \tag{5}$$

$$\alpha_i(z(k)) \geq 0, \quad \sum_{i=1}^{r} \alpha_i(z(k)) = 1 \tag{6}$$

The parameter matrices $\left(A_i, B_i \text{ and } C_i\right)$ in the local systems (2) are in non-canonical form, unlike most adaptive T–S fuzzy control literature which deals with special classes of systems whose matrices are in a canonical form. Furthermore, while from the linear local system models, a global T–S fuzzy system model can be constructed [5].
In our case, we assume that the nonlinear mathematical model is unknown with time varying parameters.

III. State feedback fuzzy control

In this section, we will present the adaptive control design for the dynamic fuzzy system presented previously.

A. Fuzzy Control Law

In the PDC design, each control rule is designed from the same rule is given as follows:

$$\begin{aligned} &R_c^i: \text{ if } z_1(k) \text{ is } M_{i1} \text{ and...and } z_p(k) \text{ is } M_{ip} \\ &\text{then} \quad u_i(k) = -K_i x(k) \qquad i = 1, 2...r \end{aligned} \tag{7}$$

where K_i is the state feedback gain of the i^{th} local model.
The global fuzzy controller is presented by:

$$\begin{aligned} u(k) &= -\frac{\sum_{i=1}^{r} w_i(z(k))K_i x(k)}{\sum_{i=1}^{r} w_i(z(k))} \\ u(k) &= -\sum_{i=1}^{r} \alpha_i(z(k))K_i x(k) \end{aligned} \tag{8}$$

by substituting (8) into (3) the global control in closed loop can be written as:

$$x(k+1) = \sum_{i=1}^{r} \alpha_i(k) \left(A_i x(k) - B_i \sum_{j=1}^{r} \alpha_j(k) K_j x(k) \right) \tag{9}$$

$$x(k+1) = \sum_{i,j}^{r} \alpha_{i,j}(k) G_{i,j} x(k) \tag{10}$$

where $G_{i,j} = A_i - B_i K_j \qquad i, j = 1..r \qquad (11)$

The fuzzy state feedback control is used to place all closed-loop eigenvalues of each local model inside the unit circle at the appropriate values in order to stabilize the plant.

B. Indirect Adaptive Fuzzy Control

The indirect adaptive fuzzy control scheme is illustrated by fig1. The gain in the control law given by equation (8) was calculated using matrice A_i and B_i whose are supposed variable during the time. Based on the assumption that the approximators are universal, we can replace A_i and B_i by their estimates A_i and B_i .

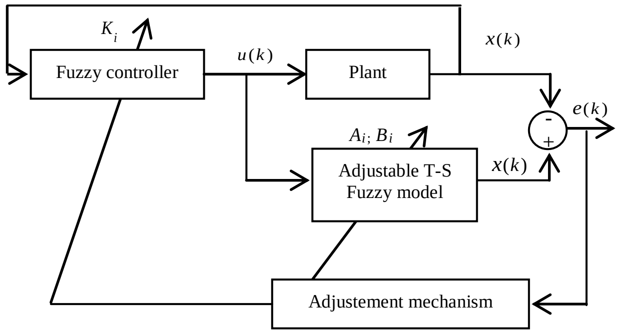

Fig. 1 Indirect adaptive fuzzy control

The error between the model and the plant is used to adjust on-line the parameters of the fuzzy model so that the error converges toward zero. We define the prediction error $e(k)$

$$\begin{aligned} e(k) &= x(k+1) - x(k+1) \\ &= A(k)x(k) + B(k)u(k) - x(k+1) \end{aligned} \tag{12}$$

Adaptation laws are determined based on the gradient method to minimize the quadratic criterion $J(k)$

$$J(k) = \frac{1}{2} e^T(k) e(k) \tag{13}$$

Then, we can deduce the following adaptation algorithm :

$$\begin{aligned} A_i(k+1) &= A_i(k) - \varepsilon_a \alpha_i(k) e(k) x^T(k) \\ B_i(k+1) &= B_i(k) - \varepsilon_b \alpha_i(k) e(k) u^T(k) \end{aligned} \tag{14}$$

where ε_a and ε_b are positive adjustment constants satisfying the stability conditions.

IV. Sability Analysis

The stability steady of the closed loop nonlinear system will be investigated in this section. For the stability analysis, a candidate Lyapunov function is proposed.

A. Convergence analysis

To demonstrate the stability of the closed-loop fuzzy control system which consists of the fuzzy model and the PDC controller, we define the following Lyapunov function:

$$V(k) = V_1(k) + V_2(k) \tag{15}$$

where $\qquad V_1(k) = \|x(k)\| \qquad (16)$

and to ensure a good estimation quality, it is necessary that these estimation errors tend asymptotically to a minimum value close to zero. The second term of the Lyapunov function is defined as follow:

$$V_2(k) = \sum_{i=1}^{r} tr\left[A_i^T(k)A_i(k)\right] + \sum_{i=1}^{r} tr\left[B_i^T(k)B_i(k)\right] \qquad (17)$$

where A_i and B_i are the estimation errors matrices, defined as:

$A_i(k) = A_i(k) - A_i(k)$ and $B_i(k) = B_i(k) - B_i(k)$

we denote that $tr\left[Q\right]$ is the trace of a matrix Q and Q^T is her transposed.

To ensure stability for the proposed scheme, the convergence conditions are obtained using the following inequalities:

$$\Delta V_1(k) < 0 \text{ and } \Delta V_2(k) < 0 \qquad (18)$$

For the first term, that concerns the dynamic fuzzy system expressed by (10), the demonstration of its stability condition is as follows:

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k) = \left\|x(k+1)\right\| - \left\|x(k)\right\| \\
&= \left\|\sum_{i,j}^{r} \alpha_{i,j}(k) G_{i,j} x(k)\right\| - \left\|x(k)\right\| \\
&\leq \left[\sum_{i,j}^{r} \alpha_{i,j}(k)\left\|G_{i,j}\right\| - 1\right]\left\|x(k)\right\| \\
&\leq \left[\sum_{i}^{r} \alpha_i(k)\lambda_{i\max} - 1\right]\left\|x(k)\right\|
\end{aligned} \qquad (19)$$

$$\Delta V_1(k) < 0 \;\text{ if }\; \sum_{i}^{r} \alpha_i(k)\lambda_{i\max} - 1 < 0 \qquad (20)$$

where $\lambda_{i\max}$ is the maximum eigenvalues of the estimates of the matrix $G_{i,j}$ for $i, j = 1..r$ given by expression (11).

$$\begin{aligned}
\Delta V_2(k) &= V_2(k+1) - V_2(k) \\
&= \sum_{i=1}^{r} tr\left[A_i^T(k+1)A_i(k+1)\right] - \sum_{i=1}^{r} tr\left[A_i^T(k)A_i(k)\right] \\
&+ \sum_{i=1}^{r} tr\left[B_i^T(k+1)B_i(k+1)\right] - \sum_{i=1}^{r} tr\left[B_i^T(k)B_i(k)\right]
\end{aligned} \qquad (21)$$

if we assume that the system parameters are slightly variable through time, we can write the following expressions:

$$\begin{aligned}
A(k+1) &= A(k) + \varepsilon_a \alpha_i(k) e(k) x^T(k) \\
B(k+1) &= B(k) + \varepsilon_b \alpha_i(k) e(k) u^T(k)
\end{aligned} \qquad (22)$$

By substituting (22) into (21), and after some manipulations using the properties of trace we obtain:

$$\Delta V_2(k) < 0 \quad \text{if} \quad 0 < \varepsilon_a < 2 \quad \text{and} \quad 0 < \varepsilon_b < 2$$

B. Theorem

Based on the Lyapunov theory for discrete systems, a sufficient stability conditions for ensuring asymptotic stability of the closed loop system (10) follows:

- $\left(\sum_{i=1}^{r} \alpha_i(k)\lambda_{i\max}\right) < 1$
- $0 < \varepsilon_a < 2$
- $0 < \varepsilon_b < 2$

V. Simulation Results

To illustrate the performance of the presented approach, two degrees of freedom robot manipulator arms system is considered as numerical example [4,9,10].

A. System description

The dynamic model shown in Fig. 2 of the system is given by the following equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u \qquad (23)$$

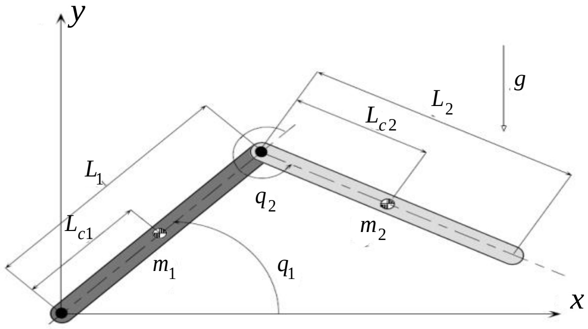

Fig. 2 Two link robot manipulator

Where q is an 2-dimensional vector of generalized coordinates representing joint positions, u an 2-dimensional control (torque) input, and $M(q)$ a symmetric positive definite inertia matrix. The terms $C(q,\dot{q})\dot{q}$ and $G(q)$ account for centrifugal/ Coriolis forces, and gravity.

where

$$M(q) = \begin{bmatrix} a_1 + 2a_3\cos(q_2) & a_2 + a_3\cos(q_2) \\ a_2 + a_3\cos(q_2) & a_2 \end{bmatrix}$$

$$C(q,\dot{q}) = a_3 \sin(q_2)\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} b_1\cos(q_1) + b_2\cos(q_1 + q_2) \\ b_2\cos(q_1 + q_2) \end{bmatrix}$$

and $a_1 = m_1 L_{c1}^2 + m_2 L_{c2}^2 + m_2 L_1^2 + I_1 + I_2$, $a_2 = m_2 L_{c2}^2 + I_2$, $a_3 = m_2 L_1 L_{c2}$, $b_1 = (m_1 L_{c1} + m_2 L_1) g$, $b_2 = m_2 L_{c2} g$

for $i = 1:2$ m_i, L_i and I_i indicate the mass, the length and the moment of inertia of arm i and L_{ci} is the distance between the joint and the center of the arm respectively. g is the constant of gravity.

m_1 and m_2 are supposed to contain unknown uncertainties with known upper bound.

Let the sate vector and the input vector at the discrete time k :

$$x(k) = \left[x_1(k)x_2(k)x_3(k)x_4(k)\right]^T = \left[q_1(k)q_2(k)\dot{q}_1(k)\dot{q}_2(k)\right]^T$$

$$u(k) = \left[u_1(k)\ u_2(k)\right]^T$$

We suppose the matrix $H(x) = M^{-1}(x) = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

And the function $F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = -M^{-1}(x)\left[C(x)+G(x)\right]$

Using the Euler approximation $X(k+1) = X(k) + Te\dot{X}(k)$, with Te is the sampling time, the dynamic system can be described by following discrete equations:

$$\begin{cases} x_1(k+1) = x_1(k) + T_e x_3(k) \\ x_2(k+1) = x_2(k) + T_e x_4(k) \\ x_3(k+1) = x_3(k) + T_e\left[f_1(x) + h_{11}(x)u_1(k) + h_{12}(x)u_2(k) \right] \\ x_4(k+1) = x_4(k) + T_e\left[f_2(x) + h_{21}(x)u_1(k) + h_{22}(x)u_2(k) \right] \end{cases}$$

The parameters of the manipulator arms are : $m_1 = m_2 = 1kg$, $L_1 = L_2 = 1m$, $L_{c1} = L_{c2} = 0.5m$, $I_1 = 5$, $I_2 = 2$ and $g = 9.8m/s^2$ and $T_e = 0.1s$ .The angular positions q_1, q_2 are constrained within $\left[-\pi, \pi\right]$.

B. Fuzzy model

The purpose, in this section, is to stabilize this system around the origin using methods presented previously.

In this simulation, the manipulator states are assumed to be measurable and the mathematical model is considered unknown and we introduce disturbances on m_1 and m_2 at the time $k = 30$. Thus, the mass variations are $\Delta m_1 = 0.4kg$ and $\Delta m_2 = 0.6kg$.

To minimize the design effort and complexity, we try to use as few rules as possible. Five fuzzy rules, for each states x_1 and x_2 , are fixed. The gaussian membership functions are shown in Fig. 3

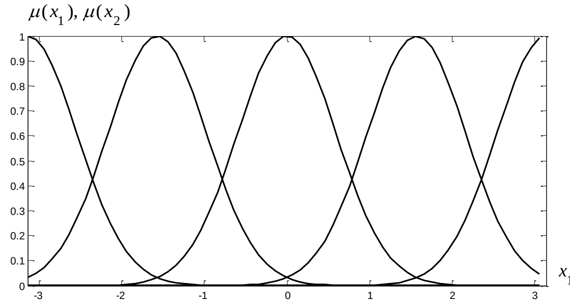

Fig. 3 Membership Functions of x_1 and x_2

The five fuzzy model rules are as following:

if $x_1(k)$ *is* M_{i1} and $x_2(k)$ *is* M_{i2} then $x(k+1) = A_i x(k) + B_i u(k)$

for $i = 1...5$

The rules of the designed fuzzy controller are:

if $x_1(k)$ *is* M_{i1} and $x_2(k)$ *is* M_{i2} then $u(k) = -K_i x(k)$

for $i = 1...5$

The feedback gains K_i are computed at each discrete time so that the closed loop poles are fixed inside the unit disc for all local models $A_i - B_i K_i$. The five poles are chosen to be :

$$p_1 = \left[-0.0114 \pm 0.2866i, \pm 0.2538i\right]$$

$$p_2 = \left[0.0016 \pm 0.3834i, -0.0022 \pm 0.5879i\right]$$

$$p_3 = \left[0.0028 \pm 0.1162i, -0.0148 \pm 0.0501i\right]$$

$$p_4 = \left[-0.2496 \pm 0.1748i, 0.2426 \pm 0.2892i\right]$$

$$p_5 = \left[0.1378, -0.42, -0.0445 \pm 0.1601i\right]$$

Fuzzy rules consequence parameters are initialized to satisfy the system controllability condition.

To adjust A_i and B_i , ε_a and ε_b have been chosen to be 0.8 and 10^{-3} respectively.

Figs. 4-6 present the simulation results of behavior of the state variables of the robot manipulator arms with parametric variation and its control laws respectively.

Simulation results demonstrate that the proposed controller is able to stabilize the robot manipulator for initial conditions $\begin{bmatrix} \frac{\pi}{6} & \frac{\pi}{4} & 0 & 0 \end{bmatrix}^T$ in the origin in spite of presence of mass variations.

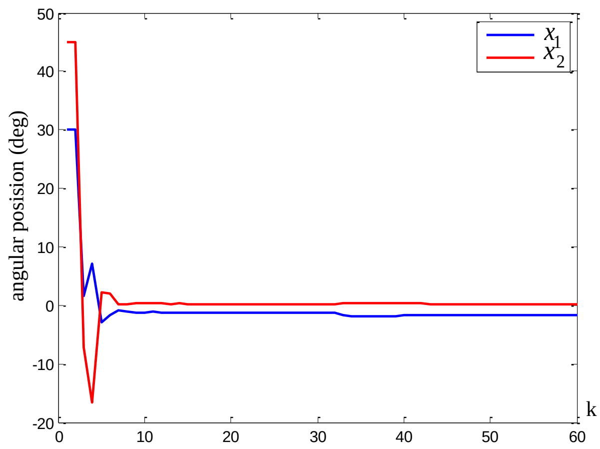

FIG. 4 Evolution of system's states x_1 and x_2 in presence of parameters variation

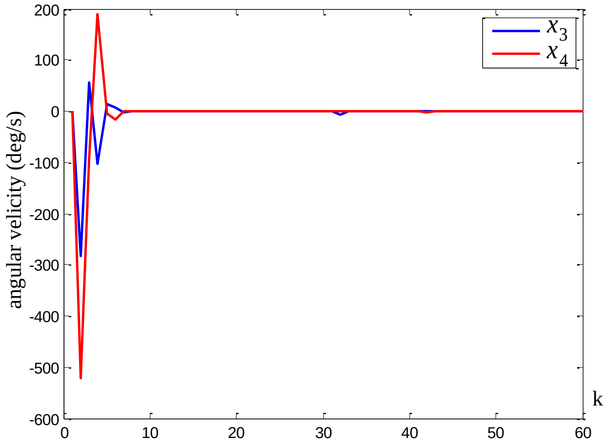

FIG. 5 Evolution of system's states x_3 and x_4 in presence of parameters variation

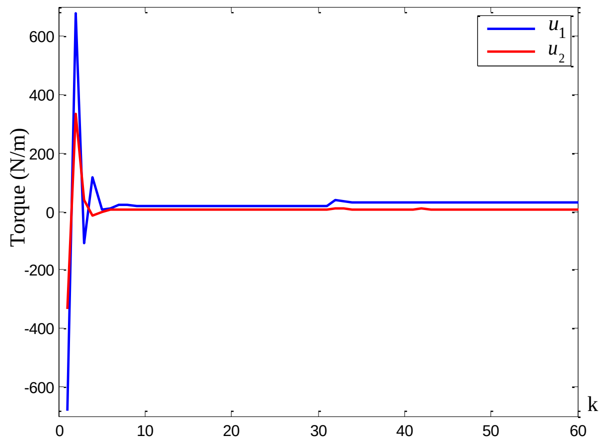

FIG. 6 Two link robot manipulator inputs in presence of parameters variation

VI. CONCLUSIONS

This work deals with indirect adaptive fuzzy control by state feed-back technique for a class of nonlinear systems. The control scheme is applied for the stabilization of unknown or/and uncertain system. At the first step, the parameters of the fuzzy model are estimated under an adaptation law. In the second step, the control law is computed basing on pole placement method. Simulation results show that the control law gives satisfactory results for the case of the time varying parameters of the systems.

REFERENCES

[1] J. J. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall Englewood Cliffs, New Jersey, 1991.

[2] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis*, John Wiley & Sons, 2001.

[3] J. T. Spooner, M. Maggiore, R. Ordbfiez , K. M. Passino, *Stable Adaptive Control and Estimation for Nonlinear Systems,* John Wiley & Sons, 2002.

[4] H.K. Khalil, *Nonlinear Control*, 1st ed, Prentice Hall, Lake Street, Upper Saddle River, 2014

[5] R. Qi, G. Tao, C. Tan, and X. Yao, "*Adaptive control of discrete-time state-space T–S fuzzy systems with general relative degree*," Fuzzy Sets and Systems, vol. 217, pp. 22–40, Nov. 2012.

[6] M. Wegmuller, J. P. von der Weid, P. Oberson, and N. Gisin, "*High resolution fiber distributed measurements with coherent OFDR*," in *Proc. ECOC'00*, paper 11.3.4, p. 109, 2000.

[7] D. Benhlima, "*Méthodologies de commande Floue adaptative de certaines classes de systèmes non linéaires*", thesis, Sfax Engineering School,Sfax, Tunisia, Mars. 2009.

[8] Y. Moraire, "*Mise en oeuvre de lois de commande pour les modèles flous de type Takagi-Sugeno*", thesis, Université de Valenciennes et du Hainaut Cambrésis, Jan. 2001.

[9] M. Manceur, "*Commande robuste des systèmes non linéaires complexes*", thesis, Université de Reims Champagne-Ardenne, Jui. 2012.

[10] C.S. Tseng, B.S. Chen, and H.J. Uang, "*Fuzzy tracking control design for nonlinear dynamic systems via TS fuzzy model Fuzzy Systems*", IEEE Transactions, p. 381 - 392, 2001.

[11] G. Feng, "*A Survey on analysis and design of model-based fuzzy control systems*", IEEE Transactions on fuzzy systems, vol. 14, n. 5, , p. 676-697, oct 2006.

[12] A.Boulkroune, and al., "*Indirect adaptive fuzzy control scheme based on observer for nonlinear systems: A novel SPR-filter approach*", Neurocomputin, 2014.

[13] J. P. Su, and al, "*Adaptive fuzzy sliding mode control with GA-based reaching laws*," Fuzzy Sets and Systems, vol. 120, pp. 145-158, 2001.

[14] M. Ratmüller and J. Murgaš, "*Fuzzy modelling and adaptive control of uncertain system*," Journal of Electrical Engineering, vol. 55, no. 9-10, pp. 251-255, 2004.