3rd International Conference on Automation, Control, Engineering and Computer Science (ACECS'16)

Proceedings of Engineering & Technology (PET)
pp. 488-496

Tracking Control of Time Delay Bilinear Systems via Block Pulse Functions

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***Abstract*—The tracking control of bilinear system with delayed state is synthesized using Block pulse functions. A linear controllers are designed allowing the systems output to follow a preshaped reference model. The parameters of the feedback regulator are derived by solving a linear algebraic equation in the least square sense. Simulation results are provided to demonstrate the merits of the proposed control approach.**

I. INTRODUCTION

Time delays systems described by functional differential equation can be utilized to model many practical physical system. This phenomenon exists widely in the transmission process of control signal, which can not be neglected in some accurate control systems. Time delays as a primary source of instability and performance degradation makes practical control systems hard to control [1]. On the other hand, bilinear system is a class of nonlinear systems that is derived by introducing the interactive product term of the state variable and the control variable in the linear state equations. The bilinear system may carry on the description for many physical systems. As a result, analysis and control of bilinear continuous time delay system have been an important topic. At present, most researchers focused on the stabilization problem [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. In some situations, the state or output of control bilinear continuous time delay system is always required to track a signal generated by a reference model. The problem of model reference tracking control is more general and difficult than the stabilization one, because the former requires the considered system not only to be stabilized but also to satisfy a specified tracking performance. Furthermore, the problem of tracking control for delayed bilinear system was not established in control literature. Then based in these observations motivate, we propose to look for a linear state feedback control with a pre-filter for bilinear system with state delayed. However, the linear state feedback control remains a simple structure that can be easily implemented in practical industrial processes.

This paper proposed a tracking control design method for bilinear system with time delay in state. The proposed control is based on linear state feedback with a pre-filter. The considered system is assumed to have a controllable linear part without time delay. The control law is aimed to ensure, not only stability but also a performance level dictated by a linear reference model whose construction is strongly inspired from the original system. Exposed mathematical developments are based on the use of block pulse functions. Among all other piecewise constant basis functions, the block-pulse functions set proved to be the most fundamental, which has the advantages to reduce computational complexities and execution time [13]. This powerful mathematical tool has been applied to solve various problems of automatic control namely, optimal control [14], [15], [16], adaptive control [20], identification [17], [18], [19], modeling and synthesis of fractional order systems[21] and model order reduction [22]. The general idea of this work consists on the projection of the state space model of the controlled system and the designed reference model over block-pulse functions basis. The useful properties of the latter tool are used to represent delay variables to convert nonlinear delay differential equations into algebraic ones. The final result is given as a linear algebraic problem which can be solved in the least square sense.

The remainder of this paper is organized as follows. A short review of Block-pulse functions is presented in the next section. The problem statement is expressed in the section 3. The proposed development is carried out, finishing with a linear algebraic system to be solved in the section 4. A numerical example is provided in the final section to illustrate the proposed method.

II. BLOCK-PULSE FUNCTIONS AND THEIR PROPERTIES

A. Block-pulse functions

We define a N-set of Block-Pulse Functions (BPF) over the interval $[0, T]$ as follows:

$$\varphi_i(t) = \begin{cases} 1 & \frac{iT}{N} \leq t \leq \frac{(i+1)T}{N}, \quad for \quad i = 1, 2, \ldots, N \\ 0 & elsewhere \end{cases} \tag{1}$$

with a positive integer value for N. Also, $\varphi_i(t)$ is the i-th Block-Pulse Functions. There are some properties for BPFs,

ISSN: 2356-5608

the most important properties are disjointness, orthogonality, and completeness.
The disjointness property can be clearly obtained from the definition of BPFs:

$$\forall \quad i,j=1,2,\ldots,N \quad \varphi_i(t)\varphi_j(t)=\begin{cases} 0, & i\neq j \\ \varphi_i(t), & i=j \end{cases}$$

The other property is orthogonality:

$$\langle \varphi_i(t), \varphi_j(t)\rangle = \begin{cases} 0, & i\neq j \\ \frac{T}{N}, & i=j \end{cases}$$

The third property is completeness. Thus, any absolutely integrable function on the time interval $[0,T]$ may be expanded on BPFs basis as follows:

$$f(t)=\sum_{i=0}^{\infty} f_i\varphi_i(t) \tag{2}$$

In practice, only N-term of (2) are considered, where N is a power of 2, that is

$$f(t)\cong\sum_{i=0}^{N-1} f_i\varphi_i(t)=F_N\phi_N(t) \tag{3}$$

with

$$F_N=\left[\begin{array}{cccc} f_0 & f_1 & \cdots & f_{N-1}\end{array}\right]$$

and

$$\phi_N(t)=\left[\begin{array}{cccc} \varphi_0(t) & \varphi_1(t) & \cdots & \varphi_{N-1}(t)\end{array}\right]^T$$

A scalar product computation leads to the values of the coefficients:

$$f_i=\frac{N}{T}\int_0^T f(t)\varphi_i(t)dt=\frac{N}{T}\int_{\left(\frac{iT}{N}\right)}^{\left(\frac{(i+1)T}{N}\right)} f(t)dt \tag{4}$$

B. Error in BPFs Approximation [24]

If we assume that $f(t)$ is a differentiable function with bounded first derivative on the time interval $[0,T]$, that is

$$\exists \bar{M}\succ 0, \quad such \quad |f'(t)|\leq \bar{M}$$

The residual error when $f(t)$ is represented in a series of BPFs over every subinterval $\left[\frac{iT}{N},\frac{(i+1)T}{N}\right]$ by the following relation:

$$\forall i\in\{0\ldots N-1\}, \quad e_i(t)=f_i\varphi_i(t)-f(t)=f_i-f(t) \tag{5}$$

The total error in the BPFs expansion of $f(t)$ may be written as

$$e(t)=\sum_{i=0}^{N-1} e_i(t), \quad t\in\left[\begin{array}{cc} 0 & T\end{array}\right] \tag{6}$$

It can be shown that [25]:

$$\|e(t)\|^2\leq \bar{M}^2\frac{T^2}{N^2} \tag{7}$$

Equation (7) clearly shows that the total error in approximation by an N-set of BPFs is $\theta\left(\frac{1}{N}\right)$. Then

$$\lim_{N\to+\infty}\|e(t)\|=0 \tag{8}$$

which establishes that we will have an exact representation of the function by using BPFs if N is high enough.

C. Operational matrix of integration

The integration matrix of the BPFs is given by [26]:

$$\int_0^t \phi_N(t)dt\cong P_N\phi_N(t) \tag{9}$$

where

$$P_N=\frac{T}{2N}\left[\begin{array}{cccc} 1 & 2 & \ldots & 2 \\ 0 & 1 & \ldots & 2 \\ \vdots & \ddots & \ddots & 2 \\ 0 & \ldots & 0 & 1 \end{array}\right]$$

D. Representation of a time delay vector function in BPFs

A vector function $x(t)$ of n dimensional components which are square integrable in $[0,T]$ can be represented approximately by a finite block pulse series

$$x(t)\cong\sum_{i=0}^{N-1} x_i\varphi_i(t)=X_N\phi_N(t) \tag{10}$$

with

$$X_N=\left[\begin{array}{cccc} x_0 & x_1 & \cdots & x_{N-1}\end{array}\right] \tag{11}$$

For an n component delay vector variable $x(t-\tau)$ with

$$x(t)=\zeta(t) \quad -\tau\leq t\leq 0 \tag{12}$$

the block pulse series approximation of $x(t-\tau)$ is given by [15]

$$x(t-\tau)\cong\sum_{i=0}^{N-1} x_i^*(\tau)\varphi_i(t)=X_N^*(\tau)\phi_N(t) \tag{13}$$

where

$$x_i^*(\tau)=\frac{N}{T}\int\limits_{\frac{iT}{N}}^{\frac{(i+1)T}{N}} x(t-\tau)dt=\begin{cases} \zeta_i(\tau) & for \quad i\prec\mu \\ x_{i-\mu} & for \quad i\geq\mu \end{cases} \tag{14}$$

with

$$\zeta_i(\tau)=\frac{N}{T}\int\limits_{\frac{iT}{N}}^{\frac{(i+1)T}{N}} \zeta(t-\tau)dt \quad for \quad i\prec\mu \tag{15}$$

and μ is the number of BPFs considered over $0\leq t\leq\tau$, and $X_N^*(\tau)=\left[\begin{array}{cccc} x_0^*(\tau) & x_1^*(\tau) & \cdots & x_{N-1}^*(\tau)\end{array}\right]$.

Let

$$\zeta_\mu(\tau)=\left[\begin{array}{cccc} \zeta_0(\tau) & \zeta_1(\tau) & \cdots & \zeta_{\mu-1}(\tau)\end{array}\right] \tag{16}$$

Then, we have [15]

$$vec(X_N^*(\tau))=E\left(n,\mu\right)vec(\zeta_\mu(\tau))+D\left(n,\mu\right)vec(X_N) \tag{17}$$

where E and D are called the shift operational matrices, given by

$$E\left(n,\mu\right)=\left[\begin{array}{c} I_{n\mu\times n\mu} \\ \cdots \\ 0_{n(N-\mu)\times n\mu}\end{array}\right] \tag{18}$$

and

$$D\left(n,\mu\right)=\left[\begin{array}{ccc} 0_{n\mu\times n(N-\mu)} & \vdots & 0_{n\mu\times n\mu} \\ \dots & \vdots & \dots \\ I_{n(N-\mu)\times n(N-\mu)} & \vdots & 0_{n(N-\mu)\times n\mu} \end{array}\right] \tag{19}$$

E. vec operator and kronecker product properties

For any matrices X, Y and Z having appropriate dimensions, the following property of the Kronecker product is given [23]:

$$vec(XYZ)=\left(Z^{T}\otimes X\right)vec(Y) \tag{20}$$

An important matrix valued linear function of a vector, denoted $mat_{(n,m)}$ was defined in [27] as follows:
If V is a vector of dimension $p=nm$ then $M=mat_{(n,m)}(V)$ is the $(n\times m)$ matrix verifying:

$$V=vec(M) \tag{21}$$

We notate e_i^p p dimensional unit vector which has 1 in the i^{th} element and zero elsewhere. The elementary matrix is defined by:

$$E_{i,j}^{p\times q}=e_i^p\otimes e_j^{q^T} \tag{22}$$

Lemma 1

Let the matrices $A=[a_{ij}]\in\mathbb{R}^{m\times n}$ and $B\in\mathbb{R}^{p\times q}$, we have:

$$\begin{aligned} & vec(A\otimes B)=vec\left(\left(\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}E_{i,j}^{m\times n}\right)\otimes B\right) \\ & =\Pi_{m,n}(B)vec(A) \end{aligned} \tag{23}$$

where

$$\begin{aligned} & \Pi_{m,n}=[vec(E_{11}^{m\times n}\otimes B)\vdots vec(E_{21}^{m\times n}\otimes B)\vdots\dots\vdots \\ & vec(E_{m1}^{m\times n}\otimes B)\vdots vec(E_{12}^{m\times n}\otimes B)\vdots vec(E_{22}^{m\times n}\otimes B)\vdots \\ & \dots\vdots vec(E_{m2}^{m\times n}\otimes B)\vdots\dots\vdots vec(E_{1n}^{m\times n}\otimes B) \\ & \vdots vec(E_{2n}^{m\times n}\otimes B)\vdots\dots\vdots\ vec(E_{mn}^{m\times n}\otimes B)] \end{aligned}$$

Lemma 2

We have:

$$\phi_N(t)\otimes\phi_N(t)=M_N\phi_N(t) \tag{24}$$

where:

$$M_N=\left[\begin{array}{c} E_{1,1}^{N^2\times N} \\ E_{2,2}^{N^2\times N} \\ E_{3,3}^{N^2\times N} \\ \vdots \\ E_{N^2,N}^{N^2\times N} \end{array}\right]$$

III. PROBLEM STATEMENT

Consider the bilinear system with time delay in state described by the following state equations:

$$\left\{\begin{array}{l} \dot{x}(t)=A_0x(t)+\bar{A}_0x(t-\tau)+\sum\limits_{i=1}^{m}A_ix(t)u_i(t) \\ \\ +\sum\limits_{i=1}^{m}\bar{A}_ix(t-\tau)u_i(t)+Bu(t) \\ \\ x(t-\tau)=x_0 \quad t\in[0,\tau] \\ \\ y(t)=Cx(t) \end{array}\right. \tag{25}$$

where $u(t)=\left[\begin{array}{cccc} u_1(t) & u_2(t) & \dots & u_m(t) \end{array}\right]^T\in R^m$ is the input vector, $x(t)\in R^n$ is the state vector, $x_0\in R^n$ is a given initial condition vector and $y(t)\in R^p$ is the output vector.
The system is assumed to be locally controllable around x_0. It can be rewrite as follows:

$$\left\{\begin{array}{l} \dot{x}(t)=A_0x(t)+\bar{A}_0x(t-\tau)+\mathcal{A}\left(u(t)\otimes x(t)\right) \\ \\ +\bar{\mathcal{A}}\left(u(t)\otimes x(t-\tau)\right)+Bu(t) \\ \\ x(t-\tau)=x_0 \quad t\in[0,\tau] \\ \\ y(t)=Cx(t) \end{array}\right. \tag{26}$$

where $\otimes$ is the symbol of the Kronecker product [23] and

$$\mathcal{A}=\left[\begin{array}{cccc} A_1 & A_2 & \dots & A_m \end{array}\right]$$

and

$$\bar{\mathcal{A}}=\left[\begin{array}{cccc} \bar{A}_1 & \bar{A}_2 & \dots & \bar{A}_m \end{array}\right]$$

The main objective of the framework is the synthesis of state feedback control with feedforward gain:

$$u(t)=\bar{N}y_c(t)-Kx(t) \tag{27}$$

where $\bar{N}\in R^{mp}$, $K\in R^{mn}$ and $y_c(t)\in R^p$ is the reference input vector. The controlled time delay bilinear system should reproduce sharply the dynamical behavior of a linear reference model and therefore responds to desired performances. Such reference model is described by the following state equations:

$$\left\{\begin{array}{l} \dot{x}_r(t)=Ex_r(t)+Fy_c(t) \\ y_r(t)=Gx_r(t) \end{array}\right. \tag{28}$$

where $x_r(t)\in R^n$ and $y_r(t)\in R^p$

IV. MAIN RESULTS

A. Linear reference model construction

The linear reference model is designed by taking the linear part without time delay of the original system, that is to say:

$$\left\{\begin{array}{l} \dot{x}_l(t)=A_0x_l(t)+Bu_l(t) \\ y_l(t)=Cx_l(t) \\ x_l(0)=x_0 \end{array}\right. \tag{29}$$

This system is assumed to be controllable and its n state components are all physically measurable. The dynamic behavior of linear system (29) could be easily tuned as desired simply with a state feedback and a feedforward gain of the following form:

$$u_l(t) = \bar{N}_0 y_c(t) - K_0 x_l(t) \tag{30}$$

The matrix $K_0 \in \mathbb{R}^{m\times n}$ can be synthesized by classical linear approaches such as pole placement. However, the matrix $\bar{N}_0 \in \mathbb{R}^{m\times p}$ which is useful to eliminate the steady state error could be determined, for $m = p$, by the following relation:

$$\bar{N}_0 = -\left[C\left[A_0 - \mathrm{B}K_0\right]^{-1}\mathrm{B}\right]^{-1} \tag{31}$$

when $p > m$, it would be possible to compute $\bar{N}_0$ through:

$$\bar{N}_0 = -\left[C_m\left[A_0 - \mathrm{B}K_0\right]^{-1}\mathrm{B}\right]^{-1} \tag{32}$$

where C_m is a restriction to m lines of C which refer to the possibly controlled outputs with accorded inputs. The last case is for $m > p$, then (31) still holds but a pseudo-inversion should be computed. Finally, The parameters of (28) are given by:

$$E = A_0 - BK_0, F = B\bar{N}_0, G = C \tag{33}$$

B. *Choose of parameters* (N, T)

For fixed reference input $y_c(t)$, the exact solution $x_r(t)$ of the constructed reference model can be obtained from the following relation:

$$x_r(t) = e^{Et}x_0 + \int_0^t e^{E(t-\tau)}Fy_c(\tau)d\tau \tag{34}$$

In order to choose the optimal number of elementary functions N of the Block Pulse Functions, we compare the exact solution $x_r(t)$ with approximate solution, given by:

$$t \in [0, T], \quad x_r(t) \cong X_{rN}\phi_N(t) \tag{35}$$

where X_{rN} denote state coefficients resulting from the scalar product (4). The time interval $[0, T]$ is given by steady state of the reference model.

C. *Control law synthesis*

From relations (26) and (27), state equation could be written as follows:

$$\begin{aligned}
\dot{x}(t) &= A_0x(t) + \bar{A}_0x(t-\tau) + \mathcal{A}\left(\left(\bar{N}y_c(t) - Kx(t)\right)\otimes x(t)\right)\\
&+\bar{\mathcal{A}}\left(\left(\bar{N}y_c(t) - Kx(t)\right)\otimes x(t-\tau)\right) + B\bar{N}y_c(t) - BKx(t)\\
&= A_0x(t) + \bar{A}_0x(t-\tau) + \mathcal{A}\left(\left(\bar{N}y_c(t)\right)\otimes x(t)\right)\\
&-\mathcal{A}\left((Kx(t))\otimes x(t)\right) + \bar{\mathcal{A}}\left(\left(\bar{N}y_c(t)\right)\otimes x(t-\tau)\right)\\
&-\bar{\mathcal{A}}\left((Kx(t))\otimes x(t-\tau)\right) + B\bar{N}y_c(t) - BKx(t)\\
&= A_0x(t) + \bar{A}_0x(t-\tau) + \mathcal{A}\left(\bar{N}\otimes I_n\right)\left(y_c(t)\otimes x(t)\right)\\
&-\mathcal{A}\left(K\otimes I_n\right)\left(x(t)\otimes x(t)\right) + \bar{\mathcal{A}}\left(\bar{N}\otimes I_n\right)\left(y_c(t)\otimes x(t-\tau)\right)\\
&-\bar{\mathcal{A}}\left(K\otimes I_n\right)\left(x(t)\otimes x(t-\tau)\right) + B\bar{N}y_c(t) - BKx(t)
\end{aligned} \tag{36}$$

The integration of equation (36) on the time interval $[0, t]$ leads to:

$$\begin{aligned}
x(t) - x(0) &= A_0\int_0^t x(\sigma)d\sigma + \bar{A}_0\int_0^t x(\sigma-\tau)d\sigma\\
&+\mathcal{A}\left(\bar{N}\otimes I_n\right)\int_0^t \left(y_c(\sigma)\otimes x(\sigma)\right)d\sigma\\
&-\mathcal{A}\left(K\otimes I_n\right)\int_0^t \left(x(\sigma)\otimes x(\sigma)\right)d\sigma\\
&+\bar{\mathcal{A}}\left(\bar{N}\otimes I_n\right)\int_0^t \left(y_c(\sigma)\otimes x(\sigma-\tau)\right)d\sigma\\
&-\bar{\mathcal{A}}\left(K\otimes I_n\right)\int_0^t \left(x(\sigma)\otimes x(\sigma-\tau)\right)d\sigma\\
&+B\bar{N}\int_0^t y_c(\sigma)d\sigma - BK\int_0^t x(\sigma)d\sigma
\end{aligned} \tag{37}$$

The expansion of state vector $x(t)$ and the fixed reference input vector $y_c(t)$ over the basis of Block-pulse functions truncated to the chosen order N given in equation (35), can be written as:

$$x(t) \cong X_N\phi_N(t), y_c(t) \cong Y_{cN}\phi_N(t) \tag{38}$$

where X_N and Y_{cN} denote state and reference input coefficients resulting from the scalar product (4).

Furthermore, the expansion of the delayed state vector $x(t-\tau)$ over the basis of Block-pulse functions truncated to the chosen order N given in equation (35), can be written as:

$$x(t-\tau) \cong X_N^*(\tau)\phi_N(t) \tag{39}$$

where $X_N^*(\tau)$ the delayed state coefficients given by equation (13).

Based on the property given by Lemma 2, the Kronecker product terms in the equation (37) can be also written as follows

$$\begin{aligned}
&y_c(t)\otimes x(t) \cong \left((Y_{cN}\phi_N(t))\otimes(X_N\phi_N(t))\right)\\
&\cong \left(Y_{cN}\otimes X_N\right)\left(\phi_N(t)\otimes\phi_N(t)\right)\\
&\cong \left(Y_{cN}\otimes X_N\right)M_N\phi_N(t)
\end{aligned} \tag{40}$$

$$x(t) \otimes x(t) \cong ((X_N\phi_N(t)) \otimes (X_N\phi_N(t))) \\ \cong (X_N \otimes X_N)(\phi_N(t) \otimes \phi_N(t)) \\ \cong (X_N \otimes X_N) M_N\phi_N(t) \tag{41}$$

$$y_c(t) \otimes x(t-\tau) \cong ((Y_{cN}\phi_N(t)) \otimes (X_N^*(\tau)\phi_N(t))) \\ \cong (Y_{cN} \otimes X_N^*(\tau))(\phi_N(t) \otimes \phi_N(t)) \\ \cong (Y_{cN} \otimes X_N^*(\tau)) M_N\phi_N(t) \tag{42}$$

and

$$x(t) \otimes x(t-\tau) \cong ((X_N\phi_N(t)) \otimes (X_N^*(\tau)\phi_N(t))) \\ \cong (X_N \otimes X_N^*(\tau))(\phi_N(t) \otimes \phi_N(t)) \\ \cong (X_N \otimes X_N^*(\tau)) M_N\phi_N(t) \tag{43}$$

The expansion of equation (37) over the considered Block-pulse functions basis yields:

$$\begin{aligned} & X_N\phi_N(t) - X_{0N}\phi_N(t) \cong \\ & A_0X_N\int_0^t \phi_N(\sigma)d\sigma + \bar{A}_0X_N^*(\tau)\int_0^t \phi_N(\sigma)d\sigma \\ & +\mathcal{A}\left(\bar{N} \otimes I_n\right)(Y_{cN} \otimes X_N) M_N\int_0^t \phi_N(\sigma)d\sigma \\ & -\mathcal{A}(K \otimes I_n)(X_N \otimes X_N) M_N\int_0^t \phi_N(\sigma)d\sigma \\ & +\bar{\mathcal{A}}\left(\bar{N} \otimes I_n\right)(Y_{cN} \otimes X_N^*(\tau)) M_N\int_0^t \phi_N(\sigma)d\sigma \\ & -\bar{\mathcal{A}}(K \otimes I_n)(X_N \otimes X_N^*(\tau)) M_N\int_0^t \phi_N(\sigma)d\sigma \\ & +B\bar{N}Y_{cN}\int_0^t \phi_N(\sigma)d\sigma - BKX_N\int_0^t \phi_N(\sigma)d\sigma \end{aligned} \tag{44}$$

The use of the integration operational matrix P_N, defined by equation (9), yields:

$$\begin{aligned} & X_N\phi_N(t) - X_{0N}\phi_N(t) \cong \\ & A_0X_NP_N\phi_N(t) + \bar{A}_0X_N^*(\tau)P_N\phi_N(t) \\ & +\mathcal{A}\left(\bar{N} \otimes I_n\right)(Y_{cN} \otimes X_N) M_NP_N\phi_N(t) \\ & -\mathcal{A}(K \otimes I_n)(X_N \otimes X_N) M_NP_N\phi_N(t) \\ & +\bar{\mathcal{A}}\left(\bar{N} \otimes I_n\right)(Y_{cN} \otimes X_N^*(\tau)) M_NP_N\phi_N(t) \\ & -\bar{\mathcal{A}}(K \otimes I_n)(X_N \otimes X_N^*(\tau)) M_NP_N\phi_N(t) \\ & +B\bar{N}Y_{cN}P_N\phi_N(t) - BKX_NP_N\phi_N(t) \end{aligned} \tag{45}$$

Simplifying the vector $\phi_N(t)$ in both sides of (45) and using the vec operator, it comes out:

$$\begin{aligned} & vec(X_N) - vec(X_{0N}) \cong \left(P_N^T \otimes A_0\right) vec(X_N) \\ & + \left(P_N^T \otimes \bar{A}_0\right) vec(X_N^*(\tau)) \\ & + \left(((Y_{cN} \otimes X_N) M_NP_N)^T \otimes \mathcal{A}\right) vec\left(\bar{N} \otimes I_n\right) \\ & - \left(((X_N \otimes X_N) M_NP_N)^T \otimes \mathcal{A}\right) vec\left(K \otimes I_n\right) \\ & + \left(((Y_{cN} \otimes X_N^*(\tau)) M_NP_N)^T \otimes \bar{\mathcal{A}}\right) vec\left(\bar{N} \otimes I_n\right) \\ & - \left(((X_N \otimes X_N^*(\tau)) M_NP_N)^T \bar{\mathcal{A}}\right) vec\left(K \otimes I_n\right) \\ & + \left((Y_{cN}P_N)^T \otimes B\right) vec(\bar{N}) - \left((X_NP_N)^T \otimes B\right) vec(K) \end{aligned} \tag{46}$$

Based on the property given by Lemma 1, it results:

$$\begin{aligned} & vec(X_N) - vec(X_{0N}) \cong \left(P_N^T \otimes A_0\right) vec(X_N) \\ & + \left(P_N^T \otimes \bar{A}_0\right) vec(X_N^*(\tau)) \\ & + \left(((Y_{cN} \otimes X_N) M_NP_N)^T \otimes \mathcal{A}\right) \Pi_{m,p}(I_n) vec\left(\bar{N}\right) \\ & - \left(((X_N \otimes X_N) M_NP_N)^T \otimes \mathcal{A}\right) \Pi_{m,n}(I_n) vec\left(K\right) \\ & + \left(((Y_{cN} \otimes X_N^*(\tau)) M_NP_N)^T \otimes \bar{\mathcal{A}}\right) \Pi_{m,p}(I_n) vec\left(\bar{N}\right) \\ & - \left(((X_N \otimes X_N^*(\tau)) M_NP_N)^T \bar{\mathcal{A}}\right) \Pi_{m,n}(I_n) vec\left(K\right) \\ & + \left((Y_{cN}P_N)^T \otimes B\right) vec(\bar{N}) - \left((X_NP_N)^T \otimes B\right) vec(K) \end{aligned} \tag{47}$$

We underline that the main idea consists on equalizing controlled system and reference model state. That is to say:

$$x(t) \cong x_r(t) \Leftrightarrow X_N\phi_N(t) \cong X_{rN}\phi_N(t) \Leftrightarrow X_N \cong X_{rN} \tag{48}$$

Furthermore, we apply the key property given by equations (13) and (17), then we have:

$$x(t-\tau) \cong X_N^*(\tau)\phi_N(t) \tag{49}$$

where

$$vec(X_N^*(\tau)) = E(n,\mu)\, vec(\zeta_\mu(\tau)) + D(n,\mu)\, vec(X_N) \tag{50}$$

with μ is the number of BPFs considered over interval $[0,\tau]$ and $E(n,\mu)$ and $D(n,\mu)$ are constant matrices given by equations (18) and (19). The constant matrix $\zeta_\mu(\tau) \in R^{n\times\mu}$ is given by

$$\zeta_\mu(\tau) = \begin{bmatrix} x_0 & x_0 & \cdots & x_0 \end{bmatrix} \tag{51}$$

Now, we replace the terms X_N by X_{rN} in equation (50), then we can compute $vec(X_N^*(\tau))$, it comes out:

$$vec(X_N^*(\tau)) = E(n,\mu)\, vec(\zeta_\mu(\tau)) + D(n,\mu)\, vec(X_{rN}) \tag{52}$$

Making use of the mat operator, given by relation (21), we can compute $X_N^*(\tau)$, it comes:

$$X_N^*(\tau) = mat(vec(X_N^*(\tau))) \tag{53}$$

By replacing in relation (47), the terms X_N by X_{rN}, we obtained the following equation, which unknows are the control law parameters

$$\begin{aligned}
& vec(X_{rN}) - vec(X_{0N}) \cong \left(P_N^T \otimes A_0\right) vec(X_{rN}) \\
& + \left(P_N^T \otimes \bar{A}_0\right) vec(X_N^*(\tau)) \\
& + \left(\left(\left(Y_{cN} \otimes X_{rN}\right) M_N P_N\right)^T \otimes \mathcal{A}\right) \Pi_{m,p}(I_n) vec\left(\bar{N}\right) \\
& - \left(\left(\left(X_{rN} \otimes X_{rN}\right) M_N P_N\right)^T \otimes \mathcal{A}\right) \Pi_{m,n}(I_n) vec\left(K\right) \\
& + \left(\left(\left(Y_{cN} \otimes X_N^*(\tau)\right) M_N P_N\right)^T \otimes \bar{\mathcal{A}}\right) \Pi_{m,p}(I_n) vec\left(\bar{N}\right) \\
& - \left(\left(\left(X_{rN} \otimes X_N^*(\tau)\right) M_N P_N\right)^T \bar{\mathcal{A}}\right) \Pi_{m,n}(I_n) vec\left(K\right) \\
& + \left(\left(Y_{cN} P_N\right)^T \otimes B\right) vec(\bar{N}) - \left(\left(X_{rN} P_N\right)^T \otimes B\right) vec(K)
\end{aligned} \tag{54}$$

The equation (54) could be written as follows:

$$\begin{aligned}
& \beta \cong \alpha_1 vec\left(\bar{N}\right) + \alpha_2 vec\left(K\right) + \alpha_3 vec\left(\bar{N}\right) + \alpha_4 vec\left(K\right) \\
& + \alpha_5 vec\left(\bar{N}\right) + \alpha_6 vec\left(K\right)
\end{aligned} \tag{55}$$

where

$$\begin{aligned}
& \beta = vec(X_{rN}) - vec(X_{0N}) - \left(P_N^T \otimes A_0\right) vec(X_{rN}) \\
& - \left(P_N^T \otimes \bar{A}_0\right) vec(X_N^*(\tau)) \\
& \alpha_1 = \left(\left(\left(Y_{cN} \otimes X_{rN}\right) M_N P_N\right)^T \otimes \mathcal{A}\right) \Pi_{m,p}(I_n) \\
& \alpha_2 = -\left(\left(\left(X_{rN} \otimes X_{rN}\right) M_N P_N\right)^T \otimes \mathcal{A}\right) \Pi_{m,n}(I_n) \\
& \alpha_3 = \left(\left(\left(Y_{cN} \otimes X_N^*(\tau)\right) M_N P_N\right)^T \otimes \bar{\mathcal{A}}\right) \Pi_{m,p}(I_n) \\
& \alpha_4 = -\left(\left(\left(X_{rN} \otimes X_N^*(\tau)\right) M_N P_N\right)^T \bar{\mathcal{A}}\right) \Pi_{m,n}(I_n) \\
& \alpha_5 = \left(\left(Y_{cN} P_N\right)^T \otimes B\right) \\
& \alpha_6 = -\left(\left(X_{rN} P_N\right)^T \otimes B\right)
\end{aligned}$$

It would be interesting to formulate our problem under the following linear algebraic equation to be solved in the least square sense:

$$\mathrm{A}\theta \cong \mathrm{B} \tag{56}$$

where

$$\mathrm{A} = \left[\begin{array}{cc} (\alpha_1 + \alpha_3 + \alpha_5) & (\alpha_2 + \alpha_4 + \alpha_6) \end{array}\right], \mathrm{B} = \beta$$

and

$$\theta = \left[\begin{array}{c} vec\left(\bar{N}\right) \\ vec(K) \end{array}\right]$$

V. Illustrative example

Let us consider a delay bilinear system given by equation (25) where:

$$A_0 = \left[\begin{array}{ccc} 0 & 1 & 0 \\ 4 & 3 & 1 \\ 1 & 0 & 1 \end{array}\right], \bar{A}_0 = \left[\begin{array}{ccc} 0 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$B = \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ 0 & 2 \end{array}\right], C = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$A_1 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0.5 & 0 \end{array}\right], A_2 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1.5 \\ 0 & 0 & 0 \end{array}\right]$$

$$\bar{A}_1 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right], \bar{A}_2 = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \end{array}\right]$$

and

$$\tau = 2, x_0 = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]^T$$

Note that the system in open loop is unstable, In order to stabilize the open loop of the linear system defined by the equation (29), we propose to place the poles of controlled system as $p_1 = -1$, $p_2 = -2$ and $p_3 = -9$. Then, the following control gains are obtained:

$$\bar{N}_0 = \left[\begin{array}{cc} 18 & 0 \\ 0 & 0.5 \end{array}\right], K_0 = \left[\begin{array}{ccc} 22 & 14 & 1 \\ 0.5 & 0 & 1 \end{array}\right]$$

The reference model defined by the equation (28) is characterized by the following parameter matrices:

$$E = \left[\begin{array}{ccc} 0 & 1 & 0 \\ -18 & -11 & 0 \\ 0 & 0 & -1 \end{array}\right], F = \left[\begin{array}{cc} 0 & 0 \\ 18 & 0 \\ 0 & 1 \end{array}\right]$$

The exact solution of the linear reference model $x_r(t)$, is given for $y_{c1}(t) = y_{c2}(t) = 1$ by

$$\left\{\begin{array}{l} x_{1r}(t) = \frac{2}{7}e^{-9t} - \frac{9}{7}e^{-2t} + 1 \\ x_{2r}(t) = \frac{18}{7}e^{-9t}\left(e^{7t} - 1\right) \\ x_{3r}(t) = 1 - e^{-t} \end{array}\right.$$

For $N = 2^7 = 128$ (Number of BPFs) and $T = 64$, it can be observed from figure (1) that we have a good approximation of the exact solution $x_r(t)$

Then, the implementation of the proposed approach leads to the following control gains

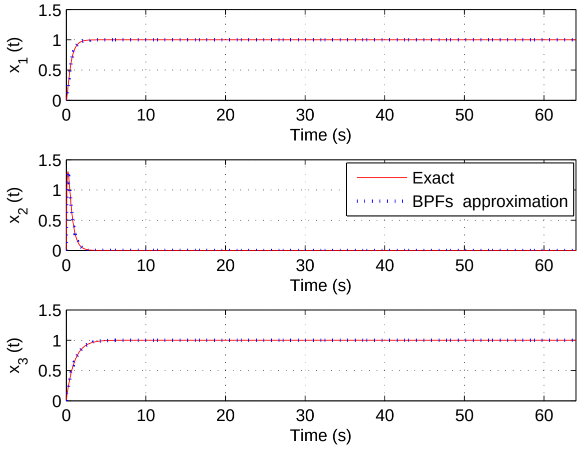

Fig. 1. States variables of the reference model

.

$$\bar{N} = \begin{bmatrix} 12.1809 & 0 \\ -1.1111 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 24.7684 & 9.9414 & -5.2539 \\ -4.9207 & -1.7868 & 3.2251 \end{bmatrix}$$

Figure (2) shows responses of the reference model and the controlled systems outputs. Simulation results prove that the proposed control law computed using the developed approach follows perfectly the desired reference model. In figure (3), shows the variation of the control signals u_1 and u_2. It appears clearly through simulation results that control objectives are attained.

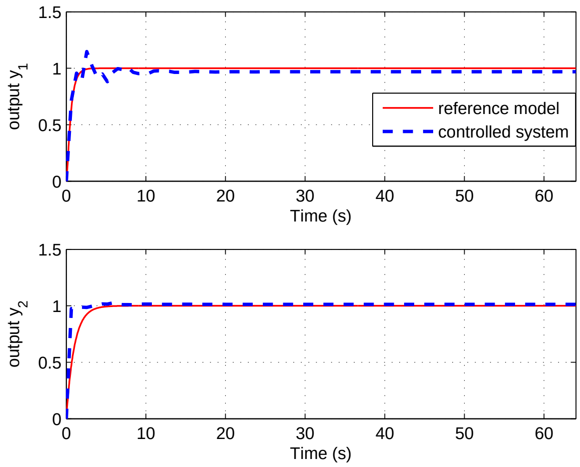

Fig. 2. Responses of the reference model and the controlled systems outputs

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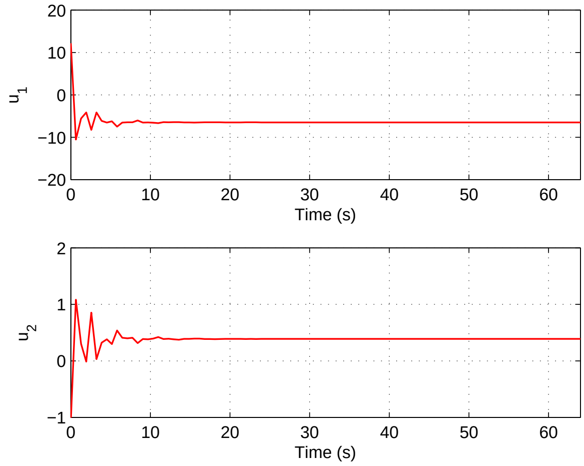

Fig. 3. control effort

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VI. Conclusion

In this paper, a new analytical approach has been introduced for the synthesis of tracking control for bilinear systems with delayed state by using Block pulse functions as an approximation tool. The useful properties of the latter tool are used to transform the differential equations into algebraic ones depending on gains of regulators. The main contribution of the paper can be summarized as the system performance guaranty jointly with stability which is obviously ensured. This is done by tracking a linear reference model. The effectiveness of the developed method is checked out by a numerical example. Simulations results obtained show clearly the accuracy of the synthesized control law. In future works, we intend to synthesize the control law for delayed nonlinear polynomial systems using orthogonal functions.

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