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On the numerical robust differentiation approaches

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***Abstract*—This paper presents an overview of important issues that arise in the design of the on-line robust differentiator. So, some numerical robust differentiation algorithms based on two well-known different approaches are studied. The first algorithm is proposed by Levant who relied on the sliding mode technique to define a finite time differentiators. The second one is a new scheme of the sliding mode differentiator. The last algorithm which is introduced by Fliess and co-authors, is based on the algebraic approach. In this paper, a comparative study between these three forms of differentiators is developed in order to discuss the strengths and weaknesses of each differentiator with fixed some specific criteria.**

***Keywords*—numerical differentiator; algebraic approach; sliding mode technique; simulations.**

I. INTRODUCTION

This paper discusses the theoretical well-studied problem of the derivatives estimation of measured signals. The large application of the state derivatives estimation leads to many research works in different fields. Some examples of these works can be cited: parameter identification [1-2], fault detection for flat systems [3], velocity and acceleration feedback for electro-pneumatic systems [4] and DC motor control [5]. The main challenge for all these applications is differentiation in the presence of noises, especially in real time where the noises are not obvious to model. In fact, it is difficult to distinguish in practice between the noise and the basic signal. Even for the usual assumption which considers that the noises correspond to the small high-frequency, the estimation of any derivative can be destroyed. Indeed, the practical differentiation is a trade-off between exact differentiation and robustness with respect to noises. The possible choices to obtain good performances of the proposed approaches deal with the problem of finding suitable linear or nonlinear filters able to reduce the effect of noise while trying to leave the based signal unchanged and without phase shift.

In the literature, the traditional approaches consist of using the finite difference method such as the Euler backward difference method. The last one is simple to implement and the most common numerical method that can be used in real time. However, this approach gives erroneous results with the sensors noises. To avoid this problem, a low-pass filter can be used to attenuate the noise on the estimated signals, but with introducing an inevitable phase delay. Other rigorous approaches consist of casting the problem of derivative estimation as an observer-design problem. In this case, the knowledge of the system/noise model is necessary. In [6-7], the proposed differentiator is a high gain observer, that is a Luenberger state observer with particular pole placement. However, to ensure an asymptotic convergence to the derivative, the observer gain should tend to infinity. So, the parameter's setting makes this method sensitive to noises or perturbations. The case of random noises/perturbations is addressed by Kalman observers whose gains are computed by the resolution of an algebraic Ricatti equation, [8]. Other example for state estimation is based on nonlinear observer theory such as a backstepping observers [9]. Unfortunately, the lack of information or an insufficient knowledge on the system dynamics makes the implementation of the linear or nonlinear state observers difficult.

To overcome this problem, some researchers looked at the synthesis of robust observers taking into account parametric uncertainties. In this case, the well-known sliding mode technique is used. In [10-11], a sliding mode observer gave interesting results. In [12], a high gain observer based on the sliding mode is proposed. In [13], the super-twisting algorithm was modified in order to observe a velocity for uncertain mechanical systems. In the other cases, the design of a differentiator is unavoidable. This problem is a classical aim in signal processing theory. To build such scheme, some features about the signal and the noise must be considered. However, in many cases the structure of the signal is unknown except from some differential inequalities. In such case, the approaches that are based on truncated Taylor series of the signal to be differentiated are potentially interesting [14]. Alternative approaches based on the sliding mode technique can also be used [15]. Among the others, a possible choice is to implement a sliding mode differentiator, as the one proposed by Levant [16]. This algorithm has a simple form and is therefore easy to be implemented. In practice, the performance of these algorithms depends on the choice of the parameter values. Indeed, these parameters depend on the input signal, according to the Lipschitz constant of the signal derivative. This constant is usually unknown accurately beforehand, especially if the signal is noisy. To avoid this problem, different research works were proposed to modify the classical 1st-order sliding mode differentiator (Super-Twisting), see for instance, [17-20].

In this paper, we are interested to study three differentiators: the sliding mode differentiator [23], the sliding mode differentiator with dynamic gains [21] and the Fliess-Mbdoup algebraic differentiator [22]. This paper is organized as follows. Section 2 introduces the basic principles of the three differentiators. In section 3, a validation phase is carried out by performing and comparing various simulation tests with different criteria. Section 4 is dedicated to discuss and to synthesize the obtained results.

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II. Differentiation Approaches

A. Sliding Modes Differentiator

As well as for the controller synthesis, the sliding modes technique shows good results in the synthesis of algorithm differentiation [16], such as the Super Twisting algorithm [23]. To design a sliding modes differentiator, let the input signal of differentiator $y(t)$ be a function defined on $[0,\infty]$ measurable in Lebesgues sense. This signal is considered as the sum of two following terms:

$$y(t)=x(t)+\xi(t) \tag{1}$$

$x(t)$ is an unknown base signal with the $(1+n)^{th}$ derivative having a known Lipschitz constant $C>0$. $\xi(t)$ is a bounded Lebesgue-measurable noise with unknown features, defined by $|\xi(t)|<\varepsilon$ with ε is sufficiently small.

In [16], Levant defines an infinite number of differentiator schemes which makes possible to estimate the n^{th} derivative of the considered signal. So, from these schemes and for $n=2$, a 2^{nd} order Sliding Modes Differentiator (2-SMD) can be defined as following:

$$\begin{cases} \dot z_0 = v_0 \\ v_0 = -\lambda_0 |s_0|^{\frac{2}{3}}\, sign(s_0) + z_1 \\ \dot z_1 = v_1 \\ v_1 = -\lambda_1 |s_1|^{\frac{1}{2}}\, sign(s_1) + z_2 \\ \dot z_2 = -\lambda_2 sign(s_2) \end{cases} \tag{2}$$

with the function $sign(.)$ is defined by:

$$sign(.)=\begin{cases} 1 & for\ \ (.)\ge 0 \\ -1 & otherwise \end{cases} \tag{3}$$

here $\lambda_0,\lambda_1,\lambda_2$ are positive gains depending on the Lipschitz constant C of $x(t)$. Here v_0, v_1 are the outputs of the differentiator. At time $t=0$, the initial values $z_0(0)=y(0)$ and $z_1(0)=z_2(0)$ were taken. Indeed, after a finite time convergence and in the absence of noise, we obtain $z_1=v_0$ the estimation of $\dot x(t)$, while $z_2=v_1$ the estimation of $\ddot x(t)$.

The parameters $\lambda_i, i\in\{0,1,2\}$ can be chosen by using the following expression (see the proof in [16]):

$$\lambda_i=\lambda_{i0}C^{\frac{1}{2-i+1}} \tag{4}$$

This expression can give an idea of these parameters' value-order. The gains $\lambda_{i0}, i\in\{0,1,2\}$ can be chosen in such a way that $\lambda_{00}<\lambda_{10}<\lambda_{20}$. However, the best way is to choose them by computer simulation .In practice, the drawback of this differentiator is the tuning up of its gains. In effect, it is not always easy to determine the gains' values for a given bandwidth of the input signal. Simple modifications of the spectral content of the input signal or its amplitude can cause a significant error in the estimation of the derivative. Moreover, the choice of algorithm parameters also presents other compromise in the presence of noise. By having relatively high values of the gains, the derivative estimation exhibit significant noise amplification. The accuracy of the 2-SMD form depends on these three gains that cannot be chosen too large so as not to differentiate the noise. The fact that this parameterization depends on the input signal is an effective limit of the method performances. In [21], a new scheme of a 2-SMD is proposed.

B. Sliding Modes Differentiator With Dynamic Gains

The accuracy of the 2-SMD depends on the choice of three parameters. So the most suitable parameters can be defined by using the adaptive mechanism, in order to regulate these gains of such an algorithm.

Let define the 2^{nd} order Sliding Modes Differentiator with Dynamic Gains (2-SMDDG) by:

$$\begin{cases} \dot z_0 = v_0 \\ v_0 = -\hat\lambda_0 |s_0|^{\frac{2}{3}}\, sign(s_0) + z_1 + \phi_0 \\ \dot z_1 = v_1 \\ v_1 = -\hat\lambda_1 |s_1|^{\frac{1}{2}}\, sign(s_1) - \hat\lambda_2 \int_{IR} sign(s_1)d\tau + \phi_1 \\ \dot z_2 = -\hat\lambda_2 sign(s_2) \end{cases} \tag{5}$$

where: $s_0=z_0-f$, $s_1=z_1-v_0$ and $s_2=z_2-v_1$ are the different sliding functions introduced in the differentiation algorithm already defined in the classic scheme (2-SMD).

$\hat\lambda_0,\hat\lambda_1$ and $\hat\lambda_2$ are the dynamic gains of the algorithm computed in real time. ϕ_0 and ϕ_1 are two adjustable terms ensuring the convergence of the new scheme of differentiator, such as:

$$\phi_0=-K_0s_0 \quad ; \quad \phi_1=-K_1s_1 \tag{6}$$

where K_0 , K_1 are two positive gains [21].

The dynamic gains $\hat{\lambda}_0, \hat{\lambda}_1, \hat{\lambda}_2$ are defined by:

$$\begin{cases} \dot{\hat{\lambda}}_0 = \left[|s_0|^{\frac{2}{3}} sign(s_0) \right] s_0 \\ \dot{\hat{\lambda}}_1 = \left[|s_1|^{\frac{1}{2}} sign(s_1) \right] s_1 \\ \dot{\hat{\lambda}}_2 = s_1 \int_0^t sign(s_1)\, dt \end{cases} \quad (7)$$

C. Algebraic differentiator

Different from the sliding modes differentiators, this algorithm is algebraic and non-asymptotic. It presents a good robustness with respect to the noises with any information and/or assumptions of their statistical properties. Consider an arbitrary smooth signal $y(t)$ which has an only property that must be C^{∞}. To estimate the time-derivative of $y(t)$ with a such algorithm, the considered signal is approximated by the classical truncated Taylor series expansion at an order N. In operational calculus notation, the derivative of the approximated signal to an $(N+1)$ order leads to this differential equation:

$$s^{N+1}\hat{y}_N(s) = s^N y(0) + s^{N-1}\dot{y}(0) + \cdots + y^{(N)}(0). \quad (8)$$

Then, the first step towards the estimation of the n^{th} derivative of y(t) is the estimation of the coefficient $y^{(N)}(0)$ from (8). The second step consists of choosing the kind of differential operator that must be used in order to estimate the desired coefficient.

Consider $y(t)$ defined as in (1) and let the linear differential operator having the following form: [22]

$$\prod\nolimits_k^{N,n} = \frac{d^{n+k}}{ds^{n+k}} \frac{1}{s} \frac{d^{N-n}}{ds^{N-n}} \quad (9)$$

with N is the truncation order, n is the index of the desired derivative ($n \le N$) and $k \in \mathbb{N}$.

This operator is chosen for its annihilator effect. One of the main advantages of this annihilator is that it provides an explicit formula for each estimate. Then, the problem due to the simultaneous estimation can be avoided. Then, the estimation is less sensitive to noise and numerical computation errors.

Considering equation (9) and for $i > n$, we have $N-n > N-i$. This provides $\frac{d^{N-n}}{ds^{N-n}} s^{-i} = 0$ and $\frac{d^{N-n}}{ds^{N-n}} s^{N-n} = (N-n)!$. This allows to conclude that the term $\frac{d^{N-n}}{ds^{N-n}}$ can annihilate all the coefficients $y^{(i)}(0)$ having $i > n$. Then, by multiplication by $\frac{1}{s}$, we obtain $(N-n)!s^{-1}y^{(n)}(0)$ and the multiplication by $\frac{d^{n+k}}{ds^{n+k}}$ is used to annihilate the coefficients having $i \prec n$.

Finally, the scheme of the algebraic time-derivative estimation is given by the following equation, [22]:

$$y_N^{(n)}(0,k,\nu) = \frac{(-1)^{n+k}(\nu+n+k)!}{(N-n)!(n+k)!T^{\nu+n+k}} \int_0^T \prod(\tau_1) y(\tau_1) d\tau_1 \quad (10)$$

where

$$\prod(\tau_1) = \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(N+1)!}{(N+i+1)!} * \sum_{j=0}^{N+k} \binom{n+k}{j} \frac{(n+1)!}{(j+1-k)!} \frac{(T-\tau_1)^{\nu+k-j-2}(-\tau_1)^{i+j}}{(\nu+k-j-2)!} \quad (11)$$

with T is the estimation window and $\nu = N+1+\mu$ where $\mu \ge 0$.

The implementation of this algorithm depends on 5 parameters' settings: N, n, T, ν and k. The higher N is, the more accurate the algebraic differentiator gets but the more complex its computing becomes. Furthermore, noise elimination of estimates is due to the presence of these iterated integrals which are computed on an estimation window T. In effect, this latter plays a key role in the implementation of the algorithm.

To illustrate the previous development, the two first derivatives are expressed. For the estimation of $\dot{y}(t)$, the parameter values are selected such as $n=1, N=3, \mu=5$ and $k=1$ and for the estimation of $\ddot{y}(t)$ are as $n=2, N=3, \mu=5$ and $k=1$. After computing and using the normalized integral to $[0,1]$ where $\tau \in [0,1]$, the following resulting equations are obtained by:

$$\tilde{\dot{y}}(0)_{N=3} = \frac{11!}{5!(4T)} (\int_0^1 [\phi_1(\tau)] y_T(\tau) d\tau \quad (12)$$

and

$$\tilde{\ddot{y}}(0)_{N=3} = \frac{-(12!)}{5!(6T^2)} (\int_0^1 [\phi_2(\tau)] y_T(\tau) d\tau \quad (13)$$

with

$$\phi_1(\tau)=\frac{-40}{336}(1-\tau)^8\tau+\frac{50}{42}(1-\tau^7)\tau^2-\frac{14}{6}(1-\tau^6)\tau^3+(1-\tau^5)\tau^4 \quad (14)$$

and

$$\phi_2(\tau)=\frac{-30}{336}(1-\tau)^8\tau+\frac{50}{42}(1-\tau^7)\tau^2-\frac{13}{6}(1-\tau^6)\tau^3+(1-\tau^5)\tau^4 \quad (15)$$

III. SIMULATION RESULTS

A. Criteria Validation

Let $y(t)=\cos(2t)+\sin(\frac{t}{2})$ the considered signal. The simulation is performed for two different tests: non-noisy test and a noisy one. For each test, there are some numbers of criteria that are chosen. For the non noisy test, the assessed criterion is the error in terms of a magnitude $|e_{\max}|$ which represents the difference between the derivative estimation and the analytical one. To have a quantitative idea on the delay provided by each differentiator, a phase shift is given in degree and computed peak to peak.

For the noisy test, we add a white Gaussian noise with zero mean and standard deviation of 0.03 to the same input signal $y(t)$ (see Fig.1).

To quantify the reduction of noise amplification's rate of these three differentiators , two indicators are used: the Signal to Noise Ratio (SNR) and the Noise Factor (NF).

The NF is defined in order to characterize the degradation rate introduced by the used algorithm, it is computed as follows:

$$\text{NF}=\text{SNR}_{\text{input}}/\text{SNR}_{\text{output}} \quad (16)$$

The SNR is a quantity measured in decibels, which gives us the power signal rate according to the noise. It is computed by the following equation:

$$\text{SNR}=10\log_{10}\left(\frac{\text{Signal energy}}{\text{Noise energy}}\right) \quad (17)$$

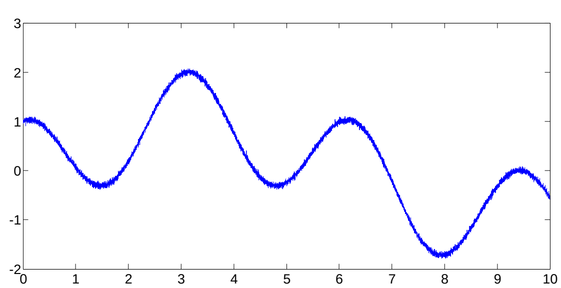

Fig. 1. Noisy input signal

So having a positive SNR means that the noise attenuation rate is high. For the low or negative values, the signal is very noisy and it is degraded. Therefore, a very high SNR value of the output corresponds to a small NF value. For all simulations, the sampling step is set at $T_e=10^{-3}$ second and for the all algebraic differentiator simulations, the size of the estimation window is set at 0.5 second

B. Simulation Tests

We present in this section some simulations to illustrate the first and the second derivatives' estimation of y(t) with three algorithms: the 2^{nd} order Sliding Modes Differentiator (2-SMD), the 2^{nd} order Sliding Modes Differentiator with Dynamic Gains (2-SMDDG), and the Algebraic Differentiator (AD).

Recall that the AD parameters are respectively: $N=3, n=1,\ \mu=5, k=1$ for the 1^{st} estimation (see equation (12)) and $N=3, n=2,\ \mu=5, k=1$ (see equation (13)) for the 2^{nd} estimation.

1) Non-Noisy Case

Without noises, note that the parameters of sliding modes differentiators are fixed such as: 2-SMD ($\lambda_0=40,\ \lambda_1=30,\ \lambda_2=10$) and 2-SMDDG ($K_0=500$, $K_1=450$).

Fig. 2 and Fig. 3 present respectively the 1^{st} and the 2^{nd} derivative estimation of the considered input signal $y(t)$ given by the studied algorithms. It is clear that all first derivative curves are very close which explains the very low values of error in terms of magnitude and phase shift (see Table I). For the second derivative, the AD, the 2-SMD and the 2-SMDDG continue to introduce a non noticeable lag with respect to the theoretical derivative.

However, Table I shows that for the estimation of the second derivative, the error given by the AD is about 6 times more important than the 2-SMD and 2 times more important than the 2-SMDDG. This AD's lack of accuracy is due to the fact that the AD's algorithm development is restricted to a such order N .

TABLE I. ERROR AND PHASE SHIFT

Algorithm	***Criteria***	***First derivative***	***Second derivative***
AD	$\lvert e_{\max}\rvert$	0.264	0.54
	Phase shift(°)	3.037	3.61
2-SMD	$\lvert e_{\max}\rvert$	0.012	0.082
	Phase shift(°)	0.17	0.91
2-SMDDG	$\lvert e_{\max}\rvert$	$2*10^{-3}$	0.24
	Phase shift(°)	0.057	0.114

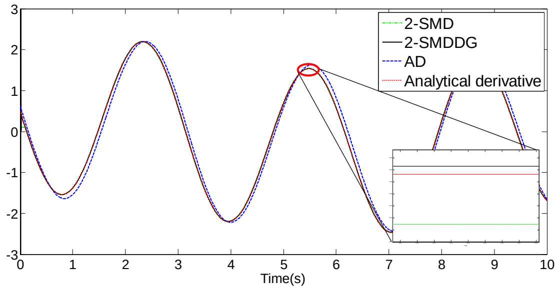

Fig. 2. Results - First order derivation

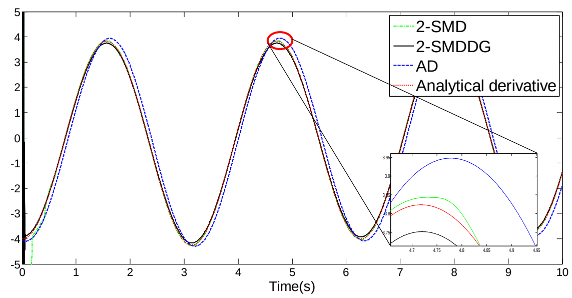

Fig. 3. Results - Second order derivation

2) Noisy Case

In the presence of the noise, the setting gains of sliding differentiator are changed as follows:

($\lambda_0 = 8,\ \lambda_1 = 7,\ \lambda_2 = 6$) for the 2-SMD and ($K_0 = 7$, $K_1 = 6$) for the 2-SMDDG.

The curves for each estimator are given in Fig.4 and Fig.5. These figures show that the 2-SMD and the 2-SMDDG estimates are clearly much more noisy than the AD estimates which is illustrated by the SNR values in Table II.

In fact, the most superior SNR values of the first derivative and the second derivative are those given by the AD. This latter has eliminated the noise amplification at the first and the second derivatives. The NF values of the AD given in table II are close to 1, which explains the robustness and the proper filtering brought by this algebraic version (see Table II).

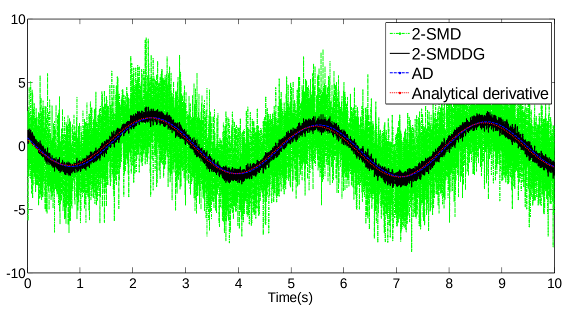

Fig. 4. Results - First order derivation

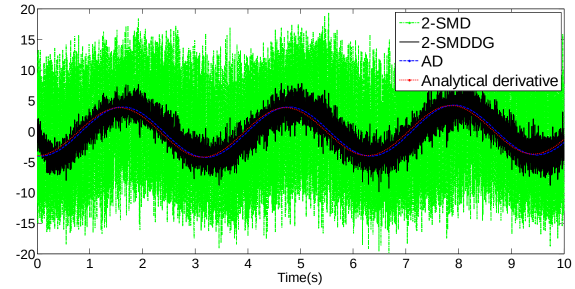

Fig. 5. Results - Second order derivation

TABLE II. SNR AND NF VALUES

Algorithm	***Criteria***	***First derivative***	***Second derivative***
AD	SNR (dB)	31.2	28.6
	NF	0.89	0.92
2-SMD	SNR (dB)	12.9	0.8
	NF	2.84	89.6
2-SMDDG	SNR (dB)	16.6	8.3
	NF	1.58	4.56

Indeed, noise amplification with the 2-SMDDG is about 2 times lower than the SMD for the estimation of the first derivative, and about 20 times smaller for the estimation of the second derivative. This noise attenuation is provided by the presence of the linear term $K_i s_i,\ i \in \{0,1\}$ in equations for each output i of the 2-SMDDG. This linear term can be seen as the equivalent command of sliding mode control laws which allows the reduction of the chattering effect.

IV. DISCUSSION

This study showed that for the 2-SMD, the higher the gains λ_i are, the faster algorithm's convergence gets. However, in case of a noisy input signal, high gain values lead to the amplification of the noise in the output signal. Indeed, the same problem is posed for the 2-SMDDG since the linear term $K_i s_i,\ i \in \{0,1\}$ allows the reduction of the chattering effect and ensures continuous smoothing in the output noise thanks to low values of convergence gains. However, if the values chosen for these gains become too low, the convergence time of the algorithm becomes slow. Therefore, for the 2-SMD and the 2-SMDDG, the choice of convergence gains remains difficult and is based on a compromise between noise reduction and convergence time of the sliding mode differentiators. The major advantage provided by the 2-SMDDG is the reduction of the noise amplification in the output signal which is done by adjusting a number of parameters less than used for the 2-SMD and the AD.(see Table III).

TABLE III. DIFFERENTIATORS' COMPARAISON

Algorithm	*Magnitude accuracy*	*Phase shift*	*Robustness*	*parameters' number*	*Time of computation*	*Ease of implementation*
AD	+	-	+++	5	-	-
2-SMD	++	++	-	3	++	+
2-SMDDG	+++	+++	+	2	+	++

One of the advantages of the AD is that it requires a unique setting parameters for both non-noisy and noisy cases. Thanks to its iterated integrals computed on an estimation window T , the AD ensures the noise elimination of estimates and occurs as the most robust differentiator studied in this paper (see Table III). Then, it is used to act as a low pass filter. Indeed, it is essential to notice that the filtering depends on the window size. Indeed, the larger the window's size is, the better filtering we get. However, a large window imposes a high truncation error. For this, it becomes necessary to make a compromise between the noise filtering and the truncation error.

The values given in Table III show that the sliding mode algorithms have a computation time much lower than the algebraic algorithms. This is due to the fact that the AD algorithm includes two "for loops" and requires as much time as compilation according to the samples number then to the sample time. Besides, the 2-SMDDG requires more time of computation than the 2-SMD since it demands the compilation of three more dynamic equations.

V. CONCLUSION AND FUTURE WORK

In this paper, two different approaches are recalled. The first is the sliding mode approach and the second is the algebraic one. In fact, two sliding mode differentiators were investigated: Sliding mode differentiator and Sliding Mode Differentiator with Dynamic Gain. This work proved that the sliding mode approach is more accurate than the algebraic one while this latter is more robust.

In future work, it is possible to improve the algebraic algorithm in order to use it in a control law instead of a physical sensor or implement it in a control loop of a physical system.

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