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Sensor fault estimation using proportional integral observer based on sliding mode principle for uncertain Takagi-Sugeno fuzzy systems

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***Abstract*—In this work, a proportional integral observer based on the sliding mode principle is used for the state and the sensor fault estimation. The state and the fault estimations are made using a particulate mathematical transformation. The application of this mathematical transformation to the initial system output let to conceive an augmented system where the initial sensor fault appears as an unknown input. An adaptive mathematical form is used for the sign function to facilitate the determination of the proportional gains of the conceived observer. The observer convergence conditions are formulated in the form of linear matrix inequalities (LMI) allowing computing the observer gains. The proposed proportional integral sliding mode observer is applied to a numerical example showing the efficiency of the fault and state estimation.**

Keywords : state estimation, Takagi-Sugeno, actuator fault, unknown input, multiple model.

I. Introduction

Many types of faults such as sensor and/or actuator faults can affect systems. To identify and eliminate these faults, many techniques have been used [13] [2] [21]. In the case of linear systems, [22] and [23] show the robustness of the fault detection and isolation (FDI) where the studied systems are affected by model uncertainties. Fault detection and isolation is used to provide invariance to uncertainty using unknown input observer (UIO) [24].

Observers with unknown inputs are used to estimate actuator faults. This estimation can be made using proportional integral observer(PIO) [7], [8]. This kind of observer is useful to estimate faults of sensors and/or actuators and uncertainty of measurement [14], [15], [17]. For fault detection and isolation, a design of a PIO is presented to minimize the L_2 gain between the uncertainty and the fault reconstructing system [16]. In addition, to obtain the desired robustness, the PIO had an additional term which is proportional to the integral of the output error estimation.

Using Takagi-Sugeno models, the authors in [13], [14] propose a method of state and sensor and actuator faults estimation using an adaptive form of proportional integral observer. The proposed technique of state and fault estimation is applied to a Takagi-Sugeno system affected simultaneously by sensor and actuator faults. For singular nonlinear systems proportional integral multi-observers are conceived showing the ability of estimation of the system state and the unknown input even in the presence of noise [10].

Some research focus in the use of adaptive observers for fault diagnosis [9], [12]. In [25], an adaptive technique of observer design for deterministic system has been developed allowing the estimation of actuator and sensor faults. The algorithm of fast adaptive fault estimation (FAFE) improve the performance of fault estimation, using constant and time-varying parameters.

The main contribution in this paper is the estimation of sensor fault using a proportional integral observer based on sliding mode principal for uncertain nonlinear systems described by multiple model structures with activation functions depending on the known system input. For sensor fault estimation, a mathematical transformation is applied to the system output. The use of this transformation allows obtaining an augmented system in which the initial sensor fault appear as an unknown input. An adaptive proportional integral sliding mode observer is conceived after that in order to estimate the state and the unknown input. A specific form of the function sign [11] is used in order to simplify the observer design. The convergence conditions of the proposed observer are computed using a quadratic Lyapunov function and are expressed as a set of linear matrix inequalities (LMI).

This paper is organized as follows, section 2 recalls the uncertain multiple model structure with unknown input. In the next section, the observer structure is presented and the stability conditions of the state and the faults estimation error are given. The section 4 presents the state estimation. An example of simulation is the subject of section 5.

II. Uncertain multiple model structure with unknown input

A nonlinear uncertain system described by multiple model structure affected by sensor fault can be written in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \\ y(t) = Cx(t) + Ef(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^r$ is the known input, $y(t) \in \mathbb{R}^p$ is the measured output, $f(t)$ represents the sensor fault and E is the fault distribution matrix. A_i, B_i and C are known constant matrices with appropriate dimensions, ΔA_i and ΔB_i are the uncertainties matrices affecting A_i and B_i, M is the number of sub-models and $\mu_i(\xi(t))$ are the activation functions which depend on the decision variable $\xi(t)$. $\xi(t)$ can be the input, the output, or the system state and must verify the following convex proprieties:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \le \mu_i(\xi(t)) \le 1 \quad (2)$$

Considering the state $z(t) \in \mathbb{R}^p$ who is a filter for the output $y(t)$ [13], [7], [14]. This state is given by the following equality :

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}_i z(t) + \bar{A}_i Cx(t) + \bar{A}_i Ef(t)) \quad (3)$$

where $-\bar{A}_i \in \mathbb{R}^{p*p}$ are stable matrices.
The augmented state $X(t) = [x^T(t) \quad z^T(t)]^T$, is introduced. This augmented state can be written as follows:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_{ai} + \Delta A_{ai})X(t) + \\ \qquad\qquad (B_{ai} + \Delta B_{ai})u(t) + E_{ai}f(t)) \\ Y(t) = C_a X(t) \end{cases} \quad (4)$$

where :

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_i C & -\bar{A}_i \end{bmatrix}, \ \Delta A_{ai} = \begin{bmatrix} \Delta A_i & 0 \\ 0 & 0 \end{bmatrix},$$
$$\Delta B_{ai} = \begin{bmatrix} \Delta B_i \\ 0 \end{bmatrix}, \ B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ E_{ai} = \begin{bmatrix} 0 \\ \bar{A}E \end{bmatrix}$$
$$\text{and} \quad C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$$

III. Observer Structure

The proposed proportional integral sliding mode observer is given in the following form :

$$\begin{cases} \dot{\widehat{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\widehat{X}(t) + B_{ai}u(t) + K_i sign(S) \\ \qquad\qquad + \alpha_{ai} + \gamma_{ai}) \\ \dot{\widehat{f}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))L_i(Y(t) - \widehat{Y}(t)) \\ \widehat{Y}(t) = C_a \widehat{X}(t) \end{cases} \quad (5)$$

where $\widehat{X}(t) \in \mathbb{R}^n$ is the estimated augmented state vector, $\widehat{Y}(t) \in \mathbb{R}^n$ is the estimated output and $K_i \in \mathbb{R}^{n-m}$ represents the gains of the i^{th} local observer. α_{ai} and γ_{ai} represent the compensation terms of the uncertainty matrices ΔA_{ai} and ΔB_{ai}.
The gains K_i , L_i , α_{ai} and γ_{ai} are added in order to ensure the asymptotic convergence to zero of the estimation error. $S \in \mathbb{R}^{n-p}$ is called sliding surface with $S = (Y(t) - \widehat{Y}(t))$.
To identify the gains K_i, the following mathematical representation of $sign(S)$ is proposed:

$$sign(S) = \frac{S}{|S|} = S - \frac{S(|S|-1)}{|S|} \quad (6)$$

In the rest of paper , the following notation is used:

$$F_i = \frac{S(|S|-1)}{|S|}$$

Using these notations, the proposed observer becomes:

$$\begin{cases} \dot{\widehat{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\widehat{X}(t) + K_i(Y(t) - \widehat{Y}(t)) + \\ \qquad\qquad B_{ai}u(t) - K_i F_i + \alpha_{ai} + \gamma_{ai}) \\ \dot{\widehat{f}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))L_i(Y(t) - \widehat{Y}(t)) \\ \widehat{Y}(t) = C_a \widehat{X}(t) \end{cases} \quad (7)$$

IV. State estimation

Lemma 1 [1]: For all matrices X and Y with appropriate dimensions, the following property is verified:

$$X^T Y + XY^T \le \beta XX^T + \beta^{-1} YY^T \quad with \quad \beta > 0 \quad (8)$$

Let us define the state estimation error $\tilde{e}(t)$ and the fault estimation error $\tilde{f}(t)$ given by the following expressions:

$$\tilde{e}(t) = X(t) - \widehat{X}(t) \quad (9)$$
$$\tilde{f}(t) = f(t) - \widehat{f}(t) \quad (10)$$

The dynamic of the state estimation can be written as follows:

$$\dot{\tilde{e}}(t) = \dot{X}(t) - \dot{\widehat{X}}(t)$$
$$\dot{\tilde{e}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_{ai} - K_i C_a)\tilde{e} + \Delta A_{ai} + \Delta B_{ai}$$
$$+ KF_i - \alpha_{ai} - \gamma_{ai} + E_a f(t)) \quad (11)$$

The dynamic of the fault estimation can be written as follows:

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \dot{\widehat{f}}(t)$$

In this work, it is supposed that $\dot{f}(t) = 0$. Under this assumption the dynamic of the fault estimation is given by the following equality:

$$\dot{\tilde{f}}(t) = -\dot{\widehat{f}}(t)$$
$$\dot{\tilde{f}}(t) = \sum_{i=1}^{M} \mu(\xi(t))(-L_i(Y(t) - \widehat{Y}\ (t))) \quad (12)$$

The variable $\varphi(t)$ given by the following expression is introduced:

$$\varphi(t) = \begin{bmatrix} \tilde{e}(t) \\ \tilde{f}(t) \end{bmatrix} \quad (13)$$

The dynamic of the state estimation error (11) and the fault estimation error (12) can be rewritten as follows :

$$\dot{\varphi}(t) = (\bar{A}_{ai}\varphi(t) + \overline{\Delta A_{ai}}X(t) + \overline{\Delta B_{ai}}u(t) + \mathrm{N}_i - \bar{\alpha}_{ai} - \bar{\gamma}_{ai}) \quad (14)$$

where :

$$\bar{A}_{ai} = \sum_{i=1}^{M}\mu_i(\xi(t))A_{mi},\ N_i = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} KF_i \\ 0 \end{bmatrix},$$

$$\overline{\Delta A_{ai}} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} \Delta A_{ai} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{\Delta B_{ai}} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} \Delta B_{ai} \\ 0 \end{bmatrix},$$

$$\bar{\alpha}_{ai} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} \alpha_{ai} \\ 0 \end{bmatrix},\ \bar{\gamma}_{ai} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} \gamma_{ai} \\ 0 \end{bmatrix},$$

$$\text{and } A_{mi} = \begin{bmatrix} A_{ai} - K_iC_a & E_{ai} \\ -L_iC_a & 0 \end{bmatrix}$$

To study the convergence to zero of the generalized estimation errors $\varphi(t)$, the Lyapunov function $V(t) = \varphi^T(t)P\varphi(t)$ is used, where P is a symmetric definite positive matrix. The generalized estimation error $\varphi(t)$ converges to zero if:

$$\dot{V}(t) = \dot{\varphi}(t)^T P\varphi(t) + \dot{\varphi}(t)P\varphi(t)^T < 0 \quad (15)$$

The derivative of the Lyapunov function $V(t)$ is given by:

$$\begin{aligned}\dot{V}(t) = (&\bar{A}_{ai}^T\varphi^T(t)P\varphi(t) + \bar{A}_{ai}\varphi(t)P\varphi^T(t) + \\ &\overline{\Delta A_{ai}}^T X^T(t)P\varphi(t) + \overline{\Delta A_{ai}}X(t)P\varphi^T(t) + \\ &\overline{\Delta B_{ai}}^T u^T(t)P\varphi(t) + \overline{\Delta B_{ai}}u(t)P\varphi^T(t) + \\ &\mathrm{N}_i{}^T P\varphi(t) + \mathrm{N}_iP\varphi^T(t) - \bar{\alpha}_{ai}^T P\varphi(t) - \\ &\bar{\alpha}_{ai}P\varphi^T(t) - \bar{\gamma}_{ai}P\varphi^T(t) - \bar{\gamma}_{ai}^T P\varphi(t))\end{aligned} \quad (16)$$

Applying (8), the dynamic of the Lyapunov function can be rewritten as follows:

$$\dot{V} = \{(\bar{A}_{ai}^T P + \bar{A}_{ai}P + \chi_i)\varphi_i{}^T\varphi_i + \theta_i\} \quad (17)$$

where :

$$\chi_i = \beta_1^{-1}\overline{\Delta A_{ai}}^T\overline{\Delta A_{ai}}P^2 + \beta_2^{-1}\overline{\Delta B_{ai}}^T\overline{\Delta B_{ai}}P^2 + \beta_3^{-1}P^2$$

$$\theta_i = \beta_1X^T(t)X(t) + \beta_2u^T(t)u(t) + \beta_3\mathrm{N}_i\mathrm{N}_i{}^T - 2\bar{\gamma}_{ai}P\varphi_i{}^T - 2\bar{\alpha}_{ai}P\varphi_i{}^T$$

$\dot{V}(t) < 0$ if the two following conditions are verified:

$$\begin{cases} \theta_i = 0 \\ \bar{A}_{ai}^T P + \bar{A}_{ai}P + \chi_i < 0 \end{cases}$$

1) First condition: $\theta_i = 0, \Longrightarrow$ the variables $\bar{\alpha}_{ai}$ and $\bar{\gamma}_{ai}$ can be given by the following inequalities:

$$\bar{\alpha}_{ai} \leq \frac{\beta_1X^T(t)X(t) + \beta_2u^T(t)u(t)}{2P\varphi_i{}^T} \quad (18)$$

$$\bar{\gamma}_{ai} \leq \frac{\beta_3\mathrm{N}_i\mathrm{N}_i{}^T}{2P\varphi_i{}^T} \quad (19)$$

2) Second Condition

$$\bar{A}_{ai}^T P + \bar{A}_{ai}P + \chi_i < 0 \quad (20)$$

The matrix $\bar{A}_{ai}$ can be rewritten as follows:

$$\bar{A}_{ai} = A_{\phi i} - K_{\phi i}C_t \quad (21)$$

where :

$$A_{\phi i} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} A_{ai} & E_a \\ 0 & 0 \end{bmatrix},$$

$$K_{\phi i} = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} K_i \\ L_i \end{bmatrix}$$

$$\text{and} \quad C_t = \sum_{i=1}^{M}\mu_i(\xi(t))\begin{bmatrix} C_a & 0 \end{bmatrix}$$

The inequality (20) can be rewritten in the following form:

$$PA_{\phi i}{}^T - PK_{\phi i}{}^T C_t{}^T + A_{\phi i}P - K_{\phi i}PC_t + \chi_i < 0 \quad (22)$$

Inequality (22) is non-linear since the term $PK_{\phi i}$ exist. To write it in the form of a linear matrix inequality, the following change of variable is used:

$$G_i = K_{\phi i}P \quad (23)$$

Inequality (22) will be as following :

$$PA_{\phi i}{}^T + A_{\phi i}P - G_iC_t - G_i^TC_t^T + \chi_i < 0 \quad (24)$$

The application of the Schur complement to (24), allows obtaining the following inequality $\forall i \in \{1...M\}$:

$$\begin{bmatrix} \upsilon_i & \overline{\Delta B}_{ai}^T P & \overline{\Delta A}_{ai}^T P & P \\ P\overline{\Delta B}_{ai} & -\beta_1^{-1}I & 0 & 0 \\ P\overline{\Delta A}_{ai} & 0 & -\beta_2^{-1}I & 0 \\ P & 0 & 0 & -\beta_3^{-1}I \end{bmatrix} < 0 \quad (25)$$

where:

$$\upsilon_i = PA_{\phi i}{}^T + A_{\phi i}P - G_iC_t - G_i^TC_t^T$$

The observer design is summarized by the following theorem:

Theorem 4.1: The system (14) describing the time evolution of the state estimation error $\tilde{x}(t)$ and the fault estimation error $\tilde{f}(t)$ is stable, if there exists a symmetric, positive definite matrix P, gain matrices $G_i, i \in \{1...M\}$ and positive scalars β_j such that the following LMIs are verified $\forall i \in \{1...M\}$:

$$\begin{bmatrix} \upsilon_i & \overline{\Delta B}_{ai}^T P & \overline{\Delta A}_{ai}^T P & P \\ P\overline{\Delta B}_{ai} & -\beta_1^{-1}I & 0 & 0 \\ P\overline{\Delta A}_{ai} & 0 & -\beta_2^{-1}I & 0 \\ P & 0 & 0 & -\beta_3^{-1}I \end{bmatrix} < 0 \quad (26)$$

where :

$$\upsilon_i = PA_{\phi i}{}^T + A_{\phi i}P - G_iC_t - G_i^TC_t^T$$

The observer gains are computed using the equation $K_{\phi_i} = G_iP^{-1}$ and the attenuation levels are given by β_i. ■

V. Simulation results

Consider the uncertain nonlinear system described by multiple model structure composed of three local models, 3 states and 3 outputs. The decision variable $\xi(t)$ is choosen as the known system input ($\xi(t) = u(t)$)

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(u(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \\ y(t) = Cx(t) + Ef(t) \end{cases} \quad (27)$$

The system matrices are:

$$A_1 = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & 0.25 & 0.25 \\ 0.25 & -0.75 & 0 \\ 0.5 & 0.25 & -2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -3 & 2 & 2 \\ 5 & -8 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 5 \\ 0.5 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.5 \\ 1 \\ 0.5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The uncertainties matrices are:

$$\Delta A_1 = \begin{bmatrix} -0.4 & 0.2 & 0.2 \\ 0.2 & -0.6 & 0 \\ 0.4 & 0.2 & -1.6 \end{bmatrix},$$

$$\Delta A_2 = \begin{bmatrix} -0.0250 & 0.0125 & 0.0125 \\ 0.0125 & -0.0375 & 0 \\ 0.0250 & 0.0125 & -0.1 \end{bmatrix}$$

$$\text{and} \quad \Delta A_3 = \begin{bmatrix} -1.5 & 1 & 1 \\ 2.5 & -4 & 0 \\ 0.25 & 0.25 & -2 \end{bmatrix}$$

The fault distribution matrix is:

$$E = \begin{bmatrix} 0 & 1 \\ 2.5 & 0.5 \\ 2 & -1.5 \end{bmatrix}$$

The weighting functions are chosen in Gaussian form and depending on the known input $u(t)$:

$$\omega_1(u(t)) = \frac{e^{\frac{(-u(t)-a_1)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}, \omega_2(u(t)) = \frac{e^{\frac{(-u(t)-a_2)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

$$\text{and} \quad \omega_3(u(t)) = \frac{e^{\frac{(-u(t)-a_3)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

where : $\sigma = 0.15$, $a_i = 0.1$, $a_i = 0.5$ and $a_i = 0.9$

The activation function are obtained after the normalization of the weighting functions: they are given by the following expressions:

$$\mu_i = \frac{\omega_i}{\sum_{j}^{3} \omega_j} \qquad \forall \quad j \in \{1..3\}$$

The fault signal $f(t)$ is given by, $f(t) = [f_1^T(t) \quad f_2^T(t)]^T$ where:

$$f_1(t) = \begin{cases} 1.5 & \text{if} \quad 20 < t < 50 \\ 1 & \text{if} \quad 90 < t < 100 \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(t) = \begin{cases} 1.5 & \text{if} \quad 20 < t < 50 \\ 1 & \text{if} \quad 50 < t < 80 \\ 0 & \text{otherwise} \end{cases}$$

The parameter $\beta_i \in \{1..3\}$ are: $\beta_1 = 29.1207$,$\beta_2 = 29.7718$ and $\beta_3 = 31.6544$. They allow guaranteeing the convergence of the estimation error to zero rather quickly.

The resolution of (26), allows finding the observer gains $\bar{K}_{m1}$,$\bar{K}_{m2}$ and $\bar{K}_{m3}$. These matrices are given below :

$$\bar{K}_{m1} = \begin{bmatrix} 1.9065 & -1.3945 & -2.5007 & 0.1097 & \ldots \\ -0.4119 & 0.7858 & 0.5956 & 0.0072 & \ldots \\ 6.3982 & -5.4822 & 0.8339 & 0.0314 & \ldots \\ 49.9361 & 0.0391 & -0.0503 & -47.3249 & \ldots \\ -0.0632 & 50.0672 & 0.1364 & 0.9804 & \ldots \\ -0.2676 & 0.2051 & 50.1091 & -1.4848 & \ldots \\ 0.9072 & -0.7665 & -0.5803 & -0.0357 & \ldots \\ -0.3740 & 0.2901 & 0.1317 & 58.8369 & \ldots \end{matrix}$$
$$\begin{matrix} \ldots & -0.0011 & 0.3190 \\ \ldots & 0.0317 & 0.0918 \\ \ldots & -0.4156 & -0.6026 \\ \ldots & 1.1404 & -1.1433 \\ \ldots & -44.5482 & 1.1869 \\ \ldots & -0.1409 & -45.6699 \\ \ldots & 146.1639 & 117.6473 \\ \ldots & 29.3351 & -88.5064 \end{bmatrix}$$

$$\bar{K}_{m2} = \begin{bmatrix} -0.4550 & -1.2742 & 1.5860 & 0.0187 & \ldots \\ 2.7729 & 0.3729 & -0.5176 & 0.0012 & \ldots \\ -0.0253 & 0.4542 & -0.1339 & 0.0059 & \ldots \\ 9.9893 & 0.0086 & -0.0140 & -8.1792 & \ldots \\ -0.0197 & 10.0192 & 0.0279 & 0.1966 & \ldots \\ -0.0605 & 0.0535 & 10.0125 & -0.2973 & \ldots \\ 0.1820 & -0.1540 & -0.1157 & -0.0036 & \ldots \\ -0.0749 & 0.0583 & 0.0259 & 11.7854 & \ldots \end{matrix}$$
$$\begin{matrix} \ldots & 0.0002 & 0.0564 \\ \ldots & 0.0058 & 0.0164 \\ \ldots & -0.0744 & -0.1074 \\ \ldots & 0.2283 & -0.2285 \\ \ldots & -7.6198 & 0.2379 \\ \ldots & -0.0294 & -7.8516 \\ \ldots & 29.2532 & 23.5351 \\ \ldots & 5.8840 & -17.7246 \end{bmatrix}$$

$$\bar{K}_{m3} = \begin{bmatrix} 2.4804 & 2.1818 & -2.1004 & 0.0300 & \ldots \\ -3.4516 & -0.5965 & 0.5907 & 0.0019 & \ldots \\ 1.4841 & -1.4016 & 0.4449 & 0.0091 & \ldots \\ 14.9763 & 0.0082 & -0.0116 & -13.0717 & \ldots \\ -0.0199 & 15.0226 & 0.0434 & 0.2954 & \ldots \\ -0.0919 & 0.0568 & 15.0455 & -0.4467 & \ldots \\ 0.2724 & -0.2299 & -0.1743 & -0.0079 & \ldots \\ -0.1126 & 0.0867 & 0.0400 & 17.6663 & \ldots \end{matrix}$$
$$\begin{matrix} \ldots & 0.0001 & 0.0892 \\ \ldots & 0.0090 & 0.0258 \\ \ldots & -0.1170 & -0.1692 \\ \ldots & 0.3423 & -0.3429 \\ \ldots & -12.2357 & 0.3564 \\ \ldots & -0.0435 & -12.5763 \\ \ldots & 43.8686 & 35.2970 \\ \ldots & 8.8139 & -26.5718 \end{bmatrix}$$

The activation functions $\mu_i(u(t))$ are presented in figure (1). This figure, shows that the used activation function verifies the convex propriety.

Simulation results are shown in figures 1, 2 and 3:

Figure (2) shows the system states and their estimations. It's clear that the proposed proportional integral sliding mode observer allows estimating well the system state. In addition,

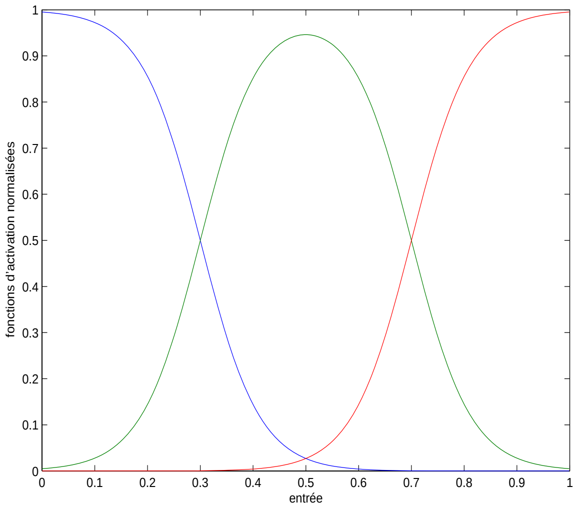

Fig. 1. Activation functions $\mu_i(u(t))$

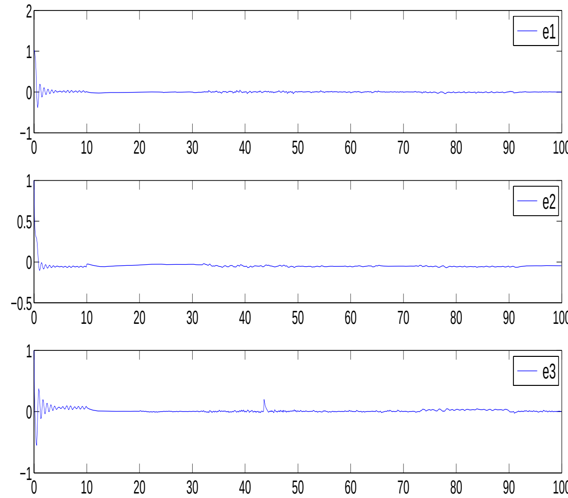

Fig. 3. State estimation error

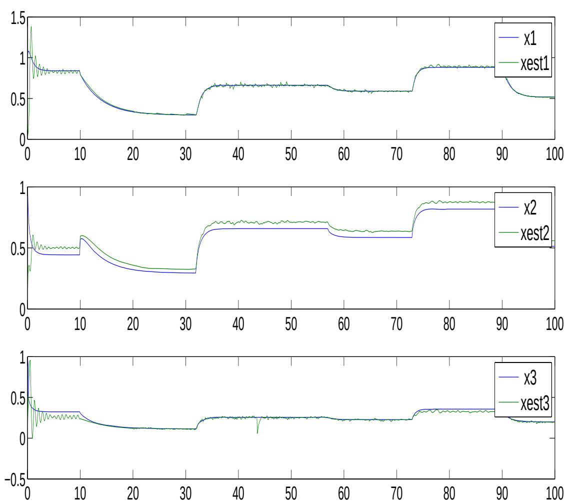

Fig. 2. States and their estimation

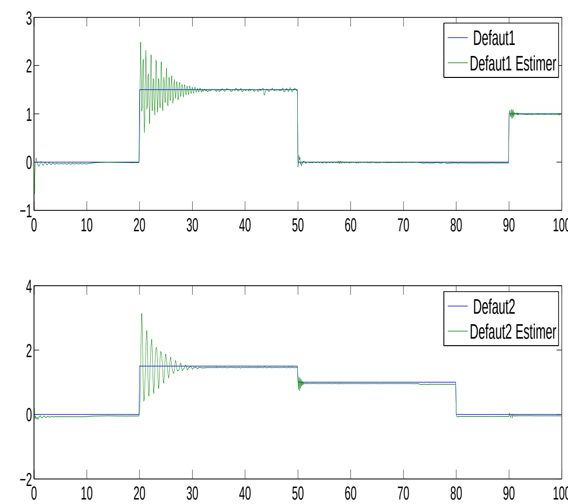

Fig. 4. Sensor fault and their estimation

this simulation shows the performance of the chosen mathematical form of the function $sign$ in order to guarantee a robustness control for state estimation. Figure (3) shows the time evolution of the state estimation error. This error is approximatively null.

Fig 4 shows the sensor faults and their estimation. It is shown that the mathematical transformation allows estimating well the sensor fault. The fault estimation follows the original signal quickly. for example, in the second part of the fault estimation figure, the first change between zeros to ones takes less to 10 second and the system is stabilized in the next change in less then 3 second and the system get the right direction.

VI. Conclusion

The developed proportional integral sliding mode observer is able to estimate the system state and the unknown inputs, its use in the case of nonlinear systems described by multiple model structures give a successful state and fault reconstruction. A specific form of the sign function is proposed and used in order to simplify the observer design the state estimation. A mathematical transformation is applied to the system output in order to conceive an augmented system in which the initial sensor fault appear as an unknown input which makes easier the fault estimation. The stability conditions of the proposed proportional integral sliding mode observer are expressed in the form of linear matrix inequalities (LMI).

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