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Chaotic Attractor With Bounded Function

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***Abstract*—In this paper we propose a new chaotic attractor with bounded behavior by combining Julia's Process with the Chua's attractor and a bounded function hyperbolic tangent. We use many forms to apply this combination and obtain different figures of these new behaviors.**

I. Introduction

During the last four decades[1],the behavior of chaotic attractor has been competently studied . It reaches many natural and artificial dynamic systems such as human heart, mechanical system, electronic circuits, etc [2]. There are so many classical attractors are known until now such us Lorenz , Rossler , Chua , Chen, and others... Our approach in this paper is to generate a new behavior of chaotic attractor using Chua attractor combined with Julia Process and a bounded function. To create a chaotic system with multi-scroll attractor became a desired goal for many engineering application [11].in order to modify a multi-scroll attractor we apply a bounded function.we choose to use hyperbolic tangent for this application and after that we will check the different modification. The rest of this paper is organized as follows. In section 2, we present the chaotic attractor and the Julia process. Then we explain the formalism mathematic . Our proposed scheme, experimental results and analysis is then presented in section 3. Finally, the paper is concluded in section4.

II. *Chaotic Attractor*

A. *Chua attractor*

In 1983, chua invented the chua circuit in order to response to two unsuccessful quest between many researchers on chaos concerning two wanting aspects of the Lorenz equations (Lorenz,1963). The existence of the chaotic attractor from the chua circuit was confirmed numerically by Matsumoto (1984) , and observed experimentally by by Zhong and Ayrom (1985), and proved strictly by Chua and al(1986). The basic approach of the proof is illustrated in a guided exercise on Chuas circuit in the well-known textbook by Hirsch, Smale and Devaney (2003).[4] This system has become one of the models in the research of chaos and is described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a(-x_1 - x_2 - x_3 + f(x_1)) \end{cases} \quad (1)$$

where $\dot{x}$,$\dot{y}$ and $\dot{z}$ are the first time derivatives and a is a real parameter. Where $f(x_1)$ is a statured function as follows :

$$f(x) = \begin{cases} k, if x > 1 \\ kx, if |x| < 1 \\ -k, if x < -1 \end{cases} \quad (2)$$

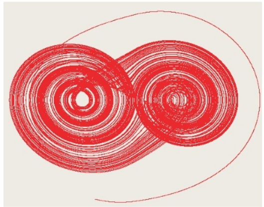

Fig. 1. classical Chua attractor with 2 scrolls

B. *julia process*

In recent years, there have been a lot of developments in Julia sets, including qualitative characters, applications and controls. In this section, the use of algorithms inspired from Julia processes, will be presented. To generate Julia processes, some of the properties are well known:

- The Julia set is a repeller.
- The Julia set is invariant
- An orbit on Julia set is either periodic or chaotic.
- All unstable periodic points are on Julia set.
- The Julia set is either wholly connected or wholly disconnected.

All sets generated only with Julia sets combination have fractal structure[13]. Real and imaginary parts of the complex numbers are separately calculated.

$$\begin{cases} x_{i+1} = x_i^2 - y_i^2 + x_c \\ y_{i+1} = x_i y_i + y_c \end{cases} \tag{3}$$

Algorithm 1 $(x_{i+1}, y_{i+1}) = P_j(x_i, y_i)$

if $x_i < 0$ **then**
  $x_{i+1} = \sqrt{(\sqrt{((x_i)^2 + (y_i)^2)} + \frac{(x_i)}{2})}$
  $y_{i+1} = \frac{y_i}{2x_{i+1}}$
end if
if $x_i = 0$ **then**
  if $x_i > 0$ **then**
    $x_{i+1} = \frac{y_i}{2y_{i+1}}$
  end if
  if $x_i < 0$ **then**
    $y_{i+1} = 0$
  end if
end if
if $x_i > 0$ **then**
  $y_{i+1} = \sqrt{(\sqrt{((x_i)^2 + (y_i)^2)} - \frac{(x_i)}{2})}$
  $x_{i+1} = \frac{y_i}{2x_{i+1}}$
  if $y_i < 0$ **then**
    $y_{i+1} = -y_{i+1}$
  end if
end if

C. *Bounded Function*

In mathematics, a function f defined on some set X with real or complex values is called bounded, if the set of its values is bounded. In other way, there exists a real number M such that

$$|f(x)| \leq M \tag{4}$$

In our case we have chosen hyperbolic tangent as a bounded function which is continuous on its domain, bounded, and symmetric, namely odd, since we have tanh(-x) = -tanh(x).

$$|tanh(x)| \leq 1 \tag{5}$$

The derivative:
[tanh(x)]' = 1/cosh2(x).

D. *Mathematic Formulation*

In this section,we study two chaotic attractor using transformation by julia's process as cited in [5]
$(y_{i+1}, x_{i+1}) = P_j(x_i, y_i)$.
Our method is to apply the methodology cited in paper [5] we conceder this two systems which are defined for two different chaotic attractor respectively:
the first chaotic attractor

$$\begin{cases} \dot{x}_1 &= f_1(x_1, x_2, x_3) + \alpha_1 \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) + \beta_1 \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) \end{cases} \tag{6}$$

We use the same chaotic attractor and we modify the value of α and β So we consider the second one as follows:

$$\begin{cases} \dot{y}_1 &= f_1(x_1, x_2, x_3) + \alpha_2 \\ \dot{y}_2 &= f_2(x_1, x_2, x_3) + \beta_2 \\ \dot{y}_3 &= f_3(x_1, x_2, x_3) \end{cases} \tag{7}$$

we treat the first system with a julia's fractal Process.
the results are obtained as follows:
$(u, v) = P_j(\dot{x}_1, \dot{x}_2)$.
$(u, v) = P_j(f_1(x_1, x_2) + \alpha_1, f_2(x_1, x_2) + \beta_1)$.
the second system as follows:
$(p, q) = P_j(\dot{y}_1, \dot{y}_2)$.
$(p, q) = P_j(f_1(x_1, x_2) + \alpha_2, f_2(x_1, x_2) + \beta_2)$.
after,we generate two outputs(X_G, Y_G)
$(X_G, Y_G) = P_j((u - p), (v - q))$.
P_j switches between two cases:

The first one when $x_{g1} > 0$ the second case is when $x_{g1} < 0$
$if\ \dot{x}_1 > 0$,the outputs are computed as follows

$$\left\{\begin{array}{lcl} u & = & \sqrt{\sqrt{\dot{x}_1^2+\dot{x}_2^2}+\frac{\dot{x}_1}{2}} \\ v & = & \frac{\dot{x}_2}{2\dot{x}_1} \end{array}\right. \quad (8)$$

$if\ \dot{x}_1 < 0$,the outputs are computed as follows

$$\left\{\begin{array}{lcl} u & = & \frac{\dot{x}_2}{2\dot{x}_2} \\ v & = & \sqrt{\sqrt{\dot{x}_1^2+\dot{x}_2^2}+\frac{\dot{x}_1}{2}} \end{array}\right. \quad (9)$$

the second system is as follow:
$(p, q) = P_j(\dot{y}_1, \dot{y}_2)$.
$if\ \dot{y}_1 > 0$,the outputs are computed as follows

$$\left\{\begin{array}{lcl} p & = & \sqrt{\sqrt{\dot{y}_1^2+\dot{y}_2^2}+\frac{\dot{y}_1}{2}} \\ q & = & \frac{\dot{y}_2}{2\dot{y}_2} \end{array}\right. \quad (10)$$

$if\ \dot{y}_1 < 0$,the outputs are computed as follows

$$\left\{\begin{array}{lcl} p & = & \frac{\dot{y}_1}{2\dot{y}_2} \\ q & = & \sqrt{\sqrt{\dot{y}_1^2+\dot{y}_2^2}+\frac{\dot{y}_1}{2}} \end{array}\right. \quad (11)$$

We concider (X_G, Y_G), the output vector generated as follows:
$(X_G, Y_G) = P_j((u - p), (v - q))$.

In this stage, we have four cases of switching:
case 1: when $(\dot{x}_1 > 0)$ and $(\dot{y}_1 > 0)$
case 2: when $(\dot{x}_1 > 0)$ and $(\dot{y}_1 < 0)$
case 3: when $(\dot{x}_1 < 0)$ and $(\dot{y}_1 > 0)$
case 4: when $(\dot{x}_1 < 0)$ and $(\dot{y}_1 < 0)$
we analyse only the first one:
when $(\dot{x}_1 > 0)$ and $(\dot{y}_1 > 0)$
so (u, v) takes this result

$$\left\{\begin{array}{lcl} u & = & \sqrt{\sqrt{\dot{x}_1^2+\dot{x}_2^2}+\frac{\dot{x}_1}{2}} \\ v & = & \frac{\dot{x}_2}{2\dot{x}_1} \end{array}\right. \quad (12)$$

and (p, q) takes this result

$$\left\{\begin{array}{lcl} p & = & \sqrt{\sqrt{\dot{y}_1^2+\dot{y}_2^2}+\frac{\dot{y}_2}{2}} \\ q & = & \frac{\dot{y}_2}{2\dot{y}_2} \end{array}\right. \quad (13)$$

so the (X_G, Y_G) takes this results

$$\left\{\begin{array}{lcl} X_G & = & \sqrt{\sqrt{(u-p)^2+(v-q)^2}+\frac{(u-p)}{2}} \\ Y_G & = & \frac{(v-q)}{2(u-p)} \end{array}\right. \quad (14)$$

III. RESULTS OF IMPLEMENTATION

In our approach we apply a combination of julia's process [5] with **chua** attractor as follow:
Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi:(f_1, f_2) \to (X_G, Y_G)$$

$$P_j(X_k, Y_k) = (X_{k+1}, Y_{k+1})$$

and we apply the hyperbolic tangent each time to a variable.

A. A combination between Julia Process and Chua attractor

The simulation results allows us to obtain a chaotic attractor in a convex space. The number of scrolls are increased as shown in the figures bellow: Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi:(f_1, f_2) \to (X_G, Y_G)$$

$$\left\{ (X_G, Y_G) = P_j((y_1 + \beta_1), (y_2 + \beta_2)). \right. \quad (15)$$

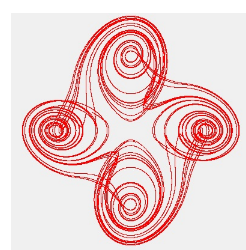

Fig. 2. CHUA attractor with 4 scrolls

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi:(f_1, f_2) \to (X_G, Y_G)$$

$$\left\{\begin{array}{r} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (X_G, Y_G) = P_j(U_1, V_1). \end{array}\right. \quad (16)$$

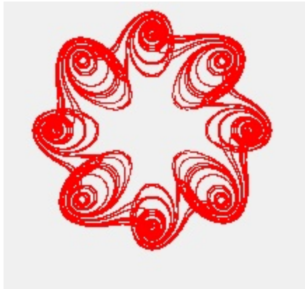

Fig. 3. CHUA attractor with 8 scrolls

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2)). \\ (U_2, V_2) = P_j(U_1, V_1). \\ (X_G, Y_G) = P_j(U_2, V_2). \end{cases} \quad (17)$$

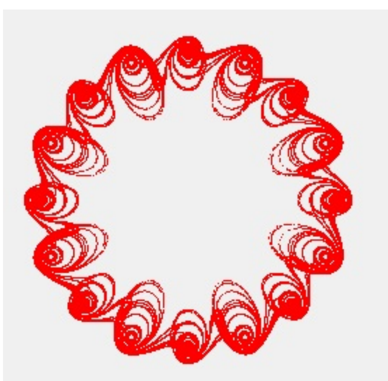

Fig. 4. CHUA attractor with 16 scrolls

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j(U_1, V_1). \\ (U_3, V_3) = P_j(U_2, V_2). \\ (X_G, Y_G) = P_j(U_3, V_3). \end{cases} \quad (18)$$

Fig. 5. CHUA attractor with 32 scrolls

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j(U_1, V_1). \\ (U_3, V_3) = P_j(U_2, V_2). \\ (U_4, V_4) = P_j(U_3, V_3). \\ (X_G, Y_G) = P_j(U_4, V_4). \end{cases} \quad (19)$$

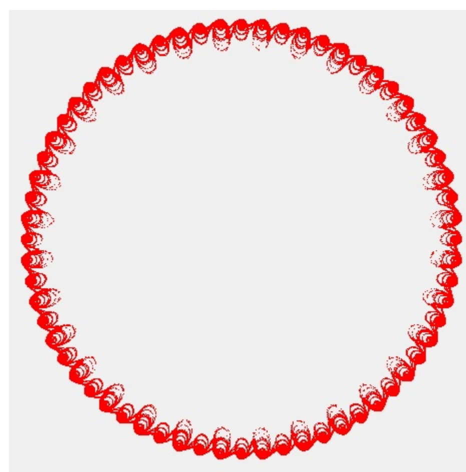

Fig. 6. CHUA attractor with 64 scrolls

*1) **Combination between Julia Process and Chua attractor with different forms**:*
In this section we present a new implementation when we combine th Chua attractor and Julia process whith constant the Figure 7bellow show us the result.
Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2)). \\ (U_2, V_2) = P_j((y_1 + \beta_1), (y_2 - \beta_2)). \\ (X_G, Y_G) = P_j((U_1 - U_2), (V_1 - V_2)). \end{cases} \quad (20)$$

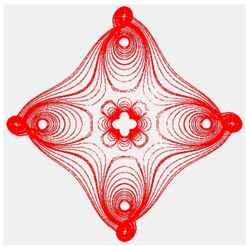

Fig. 7. CHUA attractor with 8 scrolls

B. *Application of Bounded Function*

In this section, we apply the hyperbolic tangent function in the last equation and we obtain the figures below.

*1) **Bounded function applied on the first variable**:* In the first example we apply the hyperbolic tangent as a bounded function on the first variable X. as mentioned in the equation bellow: Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j((y_1 + \beta_1), (y_2 - \beta_2). \\ (X_G, Y_G) = P_j(tanh(U_1 - U_2), (V_1 - V_2). \end{cases} \quad (21)$$

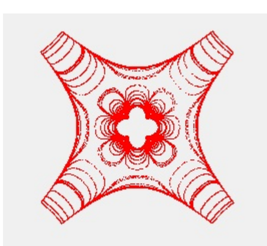

Fig. 8. Another behavior of bounded chaotic attractor applied on the first variable

*2) **Bounded function applied on the second variable**:* In the following attractor we apply the hyperbolic tangent as a bounded function on the first variable X. as mentioned in the equation bellow:

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j((y_1 + \beta_1), (y_2 - \beta_2). \\ (X_G, Y_G) = P_j((U_1 - U_2), tanh(V_1 - V_2). \end{cases} \quad (22)$$

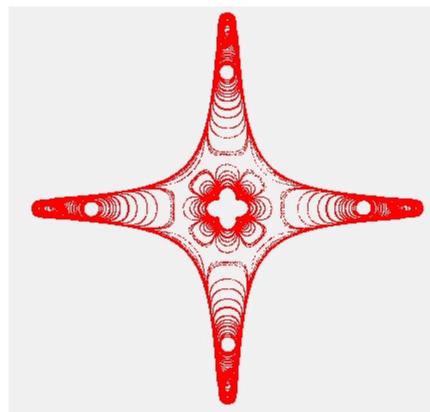

Fig. 9. chaotic attractor with bounded behavior applied on the second variable

*3) **Julia Process applied on the chaotic attractor with bounded behavior applied on the first and second variable** :*

in this attractor we have used like before the tangent hyperbolic function but we cascaded two times the julia process and the figure below shows the multiplication of the number of scrolls thanks to the julia process. Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j((y_1 + \beta_1), (y_2 - \beta_2). \\ (X_G, Y_G) = P_j(tanh(U_1 - U_2), (V_1 - V_2). \\ (X_{G1}, Y_{G1}) = P_j(X_G, Y_G). \end{cases} \quad (23)$$

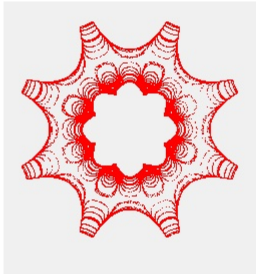

Fig. 10. Chaotic attractor with multi-scrolls bounded on the first variable

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

$$\left\{ \begin{array}{r} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2). \\ (U_2, V_2) = P_j((y_1 + \beta_1), (y_2 - \beta_2). \\ (X_G, Y_G) = P_j((U_1 - U_2), tanh(V_1 - V_2). \\ (X_{G1}, Y_{G1}) = P_j(X_G, Y_G). \end{array} \right. \tag{24}$$

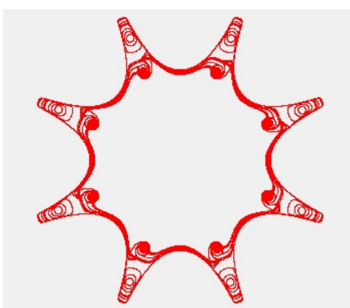

Fig. 11. Chaotic attractor with multi-scrolls bounded on the second variable

IV. Conclusion

In this paper, we have proposed a new approach to generate a new behavior of bounded chaotic attractor combined with Julia Process. the Julia Process increase the number of scrolls and the bounded function chosen here is hyperbolic tangent which limit the chaotic attractor scheme. Numerical simulations, demonstrate the validity and feasibility of proposed method. The procedure mentioned in this paper has practical application in many disciplines specially in encryption.

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