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Carbon Nanotubes dipole antenna Modeling: Comparison of electromagnetic Approach and Transmission Line Model

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***Abstract*— Carbon nanotubes are allotropes of carbon, formed by a rolled-up sheet of graphene. These are characterized by important electrical proprieties at high frequency which make them good candidates for microwaves systems such as antennas applications. The aim of this paper is related to the assessment of a current distribution along the dipole nanotubes antenna using the transmission line theory and antenna theory respectively. The dipole antenna is excited at its center by a gap voltage source. The antenna theory uses the Pocklington equations and has been solved using Galerkin method, while the transmission line approach uses the telegrapher's equations and has been solved using the transmission line technique. Some illustrative numerical results have been presented to discuss the performance of carbon nanotubes antenna compared to conventional thin wire antenna.**

***Index Terms*— Carbon nanotubes (CNT), dipole antennas, nanowire, transmission line model.**

I- INTRODUCTION

Carbon nanotubes (CNTs) were discovered by Ijima in 1991[1], there are two kinds of CNTs ; Single Wall Carbon Nanotubes (SWCNT) and Multiwalled carbon nanotubes, depending on the number of rolled up scheet of graphene. We focus here on SWCNT. Because the crystal structure of SWCNT is strongly related to that of graphene, the tubes are typically identified based on the lattice vectors of graphene.

As illustrated in Fig.1, a SWCNT can be obtained by rolling up a scheet of graphene around a vector as defined by the chiral vector $\vec{C}$ which defines the circumference of the tube. The chiral vector $\vec{C}$ is specified by a pair of integers (n,m) in the basis formed by the lattice vectors of graphene (a_1,a_2) :

$$\vec{C} = n\vec{a_1} + m\vec{a_2} \tag{1}$$

The diameter of the nanotubes can be expressed as [2]:

$$d = \frac{a}{\pi}\sqrt{n^2 + m^2 + nm} \tag{2}$$

Where ***a*** is the crystal lattice constant: $a = \sqrt{3}a_{c-c}$ (a_{c-c} =1.42 A° the distance separating two nearest atoms) [3].

We define the chiral angle, which is the angle between $\vec{C}$ and $\vec{a_1}$. The chiral angle can be evaluated as [4]:

$$\cos\theta = \frac{\vec{C}.\vec{a_1}}{|\vec{C}||\vec{a_1}|} = \frac{2n+m}{2\sqrt{n^2+nm+m^2}} \tag{3}$$

Because the honeycomb lattice is symmetric, the value of θ is in the range $0 \le \theta \le 30°$.

If the nanotubes is of the type (n, 0) (θ=0), is called zigzag nanotubes, nanotubes of the type (n,n) (θ=0), is called armchair nanotubes. Both zigzag and armchair nanotubes is called achiral tube (n,m≠n≠0) .

There are two possible choices for the pair of the integers (n,m): if the condition *n-m=3l* is always satisfied, the nanotubes is metallic , else if the condition n-m=3l+1 is satisfied, the nanotubes is semiconducting [5].

In our previous modeling work [6], CNT is considered as antennas, but did not discuss their performance potential. Recently, SWCNTs are synthesized with a length comparable to the microwave in free space. This motivates our work to explore their properties as antenna. Since CNT can be grown having length on the order of centimeter and can be metallic. In the range of centimeter and millimeter applications, CNT antennas are originally proposed by Burke [7].

In [7] CNT dipole antennas is modeling based on a transmission line approach, so the transmission line parameters; kinetic inductance $\boldsymbol{L_K}$, quantum capacitance $\boldsymbol{C_Q}$ and resistance R are determined.

Another common approach for simulating electromagnetic wave propagation along CNT based on these electrodynamics properties is proposed [3]. This approach presents a macroscopic view for the interaction of high frequency electromagnetic field with CNTs based on equivalent dynamic surface conductivity.

In this paper, fundamental properties of finite length dipole CNT antennas are investigated using the transmission line approach and a Hallén's integrodifferential equation respectively. The current distribution and the radiation pattern are presented and compared to the classical metallic antennas of some size and shape.

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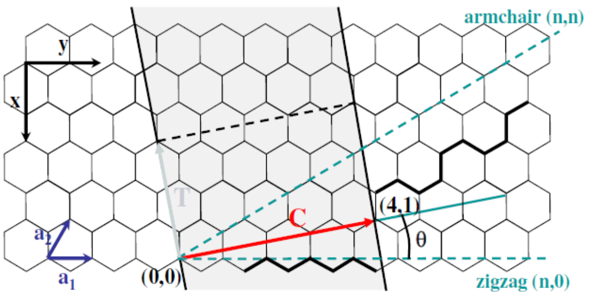


Fig.1: crystallographic graphene structure showing coordinate system, lattice vectors, chiral vector for different type of CNT.

II- MATHEMATIC FORMULATION

1- Electron fluid model of CNT transmission line

Electron fluid model is presented to describe the linear response of a SWNT to an applied electromagnetic field. This method is based on the classical problem of moving point charges in electric field and using the effective mass for the moving electrons to include the effect of CNT lattice. The CNT is modeled as a continuous infinitesimally cylinder of length ***l*** and radius ***r*** disposed along the z axis. The thermodynamic equilibrium is characterized by uniform distribution of π-electrons. This equilibrium distribution is perturbed by an applied electromagnetic field. The motion of the perturbed π-electrons is modeled as a compressible charged fluid with friction. Note that $V_Z(r,t)$ is the velocity of the electron fluid, and a point at the surface S is identified by the position vector ***r***. $\boldsymbol{n_0}$ is the density of electron fluid in equilibrium, $\boldsymbol{n=n_0+\delta n(r,t)}$ is the surface number density of electron fluid. $\boldsymbol{p=p_0+\delta p(r,t)}$ is the pressure of the fluid, where $\boldsymbol{p_0}$ is the equilibrium pressure. The variation of pressure perturbation is related to the variation of electron density by the relation: $\delta p = m_{eff} v_F^2 \delta n$ where m_{eff} is the mean effective mass of electrons and $\boldsymbol{v_F}$ is the electron Fermi velocity and equal to $\boldsymbol{8.10^5 m.s^{-1}}$. The displacement of electron fluid obeys to the law of momentum conservation which can be expressed as[8]:

$$n_0 m_{eff} \frac{\partial V_Z}{\partial t} + v n_0 m_{eff} V_Z + \frac{\partial \delta p}{\partial z} = n_0 e E_Z \qquad (4)$$

This momentum conservation equation can be expressed used the longitudinal current $\boldsymbol{I_Z=2\pi.r.e.n_0.V_Z}$ and the surface charge density $\boldsymbol{q=2\pi.r.e.n}$

$$\frac{\partial I_Z}{\partial t} + v I_Z + v_F^2 \frac{\partial q}{\partial z} = \frac{2\pi r n_0 e^2}{m_{eff}} E_Z \qquad (5)$$

$$L_K \frac{\partial I_Z}{\partial t} + R I_Z + \frac{1}{C_Q}\frac{\partial q}{\partial z} = E_Z \qquad (6)$$

Where the kinetic inductance, ohmic resistance and quantum capacitance per unit length are respectively given by:

$$L_K = \frac{h}{8 v_F e^2}\ ;\ R = \frac{vh}{8 v_F e^2} \text{ and } C_Q = \frac{8e^2}{v_F h}$$

The CNT can be modeled by an equivalent circuit with series elements distributed (Fig.2).

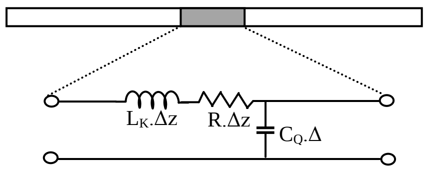


Fig.2: Circuit model for electron flow along CNT.

2- Circuit model for two CNTs

In the case of two CNTs above a metallic ground plane as shown in Fig.3, the equivalent circuit model would be the combination of the equivalent circuit for electron current flow along the two CNTs and the conventional equivalent circuit model based on electrostatic capacitance and magnetic inductance [9].

The electrostatic capacitance can be expressed by [4]:

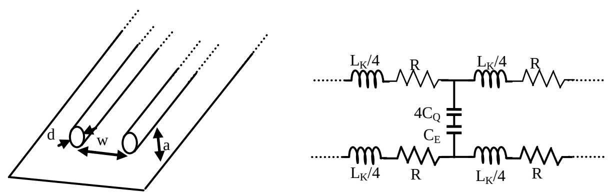
$$C_{ES} = \frac{2\pi\varepsilon_D}{\cosh^{-1}(2a/d)} \approx \frac{2\pi\varepsilon_D}{\ln(d/a)} \qquad (7)$$

The magnetic inductance is given by [4]:

$$L_M = \frac{\mu_M}{2\pi}\cosh^{-1}\left(\frac{2a}{d}\right) \approx \frac{\mu_M}{2\pi}\ln\left(\frac{a}{d}\right) \qquad (8)$$

By a simple calculation we prove that:

$$\frac{L_M}{L_k} \sim 10^{-4} \quad \text{and} \quad \frac{C_{ES}}{C_Q} \sim 1 \qquad (9)$$



(a) (b)

Fig.3: (a) Geometry of two CNTs over ground plane, (b) equivalent circuit model for two CNTs over ground plane.

By applying ohm's law we find

$$\begin{cases} \dfrac{\partial^2 V(x)}{\partial x^2} - \gamma_p^2 . V(x) = 0 \\ \dfrac{\partial^2 I(x)}{\partial x^2} - \gamma_p^2 . I(x) = 0 \end{cases} \qquad (10)$$

Where γ_p is the propagation constant, and can be calculated as:

$$\gamma_p^2 = 2\left(R + jw\frac{L_k}{4}\right)(jC_T w) \qquad (11)$$

The current distribution can be obtained from the differential equation:

$$I(z)=\begin{cases}\frac{V_0^+}{Zc}\sinh\left[\gamma_p(\frac{L}{2}-z)\right]; si\,0<z<\frac{L}{2}\\ \frac{V_0^+}{Zc}\sinh\left[\gamma_p(\frac{L}{2}+z)\right]; si-\frac{L}{2}<z<0\end{cases} \quad (12)$$

The characteristic impedance is given by:

$$Z_c=\frac{1}{\sqrt{2}}\sqrt{(R+iw\frac{L_k}{4})/iwC_{tot}} \quad (13)$$

In the case where no loss the current distribution can be expressed by:

$$I(z)=\begin{cases}\frac{V_0^+}{Zc}\sin\left[k_p(\frac{L}{2}-z)\right]; si\,0<z<\frac{L}{2}\\ \frac{V_0^+}{Zc}\sin\left[k_p(\frac{L}{2}+z)\right]; si-\frac{L}{2}<z<0\end{cases} \quad (14)$$

3- Radiation property

The electric field is related to the current distribution by the expression:

$$E_\theta=i\eta\frac{ke^{-ikr}}{4\pi r}\sin\theta\left[\int_{-l/2}^{l/2}I(z)e^{ikz\cos\theta}dz\right] \quad (15)$$

By substituting equation (14) in equation (15) we find:

$$E_\theta=i\eta\frac{k}{k_p}\frac{I_0e^{-ikr}}{2\pi r}\sin\theta\frac{\cos(\frac{kl}{2}\cos\theta)-\cos(\frac{k_pl}{2})}{1-(\frac{k}{k_p})^2\cos^2\theta} \quad (16)$$

The radiation intensity is related to the Pointing vector by:

$$U=r^2\left\|\vec{\Pi}\right\|=\left(\frac{k}{k_p}\right)^2\frac{I_0^2\eta}{8\pi^2}\left[\sin\theta\frac{\cos(\frac{kl}{2}\cos\theta)-\cos(\frac{k_pl}{2})}{1-(\frac{k}{k_p})^2\cos^2\theta}\right]^2 \quad (17)$$

4- Dynamic conductivity and integrals equations for CNT dipole antenna

Dynamic conductivity of CNT represents a macroscopic quantity relating to the perturbation of electron flow along the CNT due to the temporal variation of the applied electric field along it. Dynamic conductivity of CNT can be calculated using the Boltzmann kinetic equation of CNT.

For a small radius of CNT, the dynamic conductivity can be expressed as [10]:

$$\sigma_{cn}(w)=\sigma_{zz}(w)\simeq-j\frac{2e^2v_F}{\pi^2\hbar(w-j\nu)} \quad (18)$$

Where ***e*** is the electron charge, ***v*** is the relaxation electron frequency for CNT (equal to $3.10^{-12}s^{-1}$), ***a*** is the CNT radius, $\hbar$ the reduced Planck constant and here we use the Fermi velocity as v_F=9,71.10^5 m/s.

Figure 4 shows the dynamic conductivity variation as function of frequency for different radius values of CNT. The dynamic conductivity increases when the radius decreases that shows the important conductivity of CNT with small radius.

This equivalent surface conductivity is characterized by a complex value with a negative imaginary part. This negative imaginary part represents an inductive effect in CNT. This inductive effect introduces a deceleration in the electromagnetic wave velocity along the CNT which corresponds to decreasing the wavelength. This property is so important in the passive RF devices and antennas.

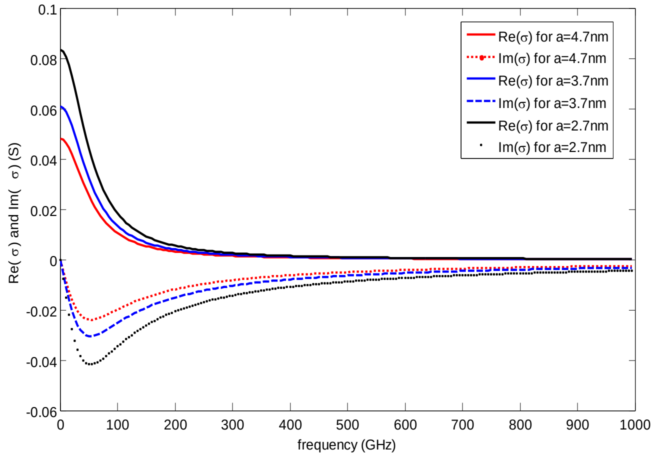


Fig.4: Dynamic conductivity as function of frequency for CNT for different radius values.

CNT antennas can be treated as a finite conductivity cylinders excited at its center by a slice-gap source of unit voltage. In this case we use the dynamic conductivity to include the electrical property of the CNT in the mathematic formulation. For the case of simple CNT dipole antenna oriented along the *z* axis, current density is given by the ohm's law:

$$J_z(z)=\sigma(w)E(z) \quad (19)$$

Where the electrical field is the sum of incident and radiated electrical field:

$$E(z,w)=E^{in}(z,w)+E^r(z,w) \quad (20)$$

The radiated electrical field can be expressed as [3]:

$$E_z^r=\frac{1}{j4\pi w\varepsilon}(k^2+\frac{\partial^2}{\partial z^2})\int_{-h}^{h}\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}}I(z')dz' \quad (21)$$

Where *h* is the half length of the dipole and *a* is the antenna radius which will be in the order of nanometer, then we can approximate the current density as: $I(z)=2\pi aJ(z)$

So we can rewrite the Ohm's law as:

$$\frac{I(z)}{2\pi a}=\sigma_{cn}(E_z^{in}+E_z^{sc}) \quad (22)$$

Finally, we can deduce the Hallen's integral equation [3]:

$$(k^2+\frac{\partial^2}{\partial z^2})\int_{-h}^{h}R(z-z')I(z')dz'=-j4\pi w\varepsilon E_z^{in}(z) \quad (23)$$

Where the function R is given by:

$$R(z-z')=\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}}+\frac{w\varepsilon}{a\sigma}\frac{e^{-jk|z-z'|}}{k} \quad (24)$$

The antenna is divided into N=2M+1 segments of width $\Delta = \frac{L}{N}$, as shown in fig.5, and (23) is evaluated used the delta function basis for the small sample of antenna as:

$$(k^2 + \frac{\partial^2}{\partial z^2})V(z_n) = 2kE_z^{in}(z_n) \tag{25}$$

Where $$V(z_n) = \frac{j\eta}{2\pi}\int_{-h}^{h} R(z_n - z')I(z')dz' \tag{26}$$

And $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

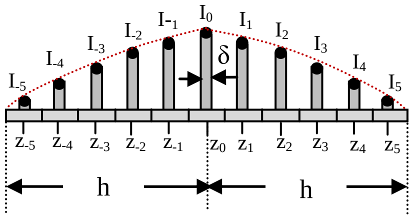


Fig.5: Segmentation of antenna with a delta-function (N=11)

We can apply a finite difference approximation to the second derivative in z, which allows rewrite equation (25) as follows:

$$k^2 V_n + \frac{V_{n+1} - 2V_n + V_{n-1}}{\Delta^2} = 2kE_n^{in} \tag{27}$$

Denoting that $V(z_n) = V_n$ and $E^{in}(z_n) = E_n^{in}$

Then we have: $$V_{n+1} - 2\alpha V_n + V_{n+1} = dE_n \tag{28}$$

Where: $\alpha = 1 - \frac{k^2\Delta^2}{2}$; $d = 2k\Delta^2$ and $-(M-1) \le n \le M-1$

We can convert (35) in the corresponding matrix equation, as it's shown here for M=3.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2\alpha & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2\alpha & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2\alpha & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2\alpha & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2\alpha & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{-3} \\ V_{-2} \\ V_{-1} \\ V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = d. \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{-3} \\ E_{-2} \\ E_{-1} \\ E_0 \\ E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{29}$$

We may write (29) as: $AV = d\delta E$ where δ is the projection matrix, we note that P is its complement $P = I_d - \delta$, which enforce the edges conditions $I_{\pm M} = 0$.

$$PI = (I_d - \delta)I = 0 \tag{30}$$

The current along the CNT is expressed as the sum of the samples current I_m using the basis function B(z):

$$I(z) = \sum_{m=-M}^{M} I_m B(Z - Z_m) \tag{31}$$

Then equation (33) can be rewritten as follows:

$$V(z_n) = \sum_{m=-M}^{M} I_m \frac{j\eta}{2\pi}\int_{-h}^{h} R(z_n - z_m - z)B(z)dz = \sum_{m=-M}^{M} Z_{nm} I_m \tag{32}$$

With the impedance matrix is given by:

$$Z_{nm} = \frac{j\eta}{2\pi}\int_{-h}^{h} R(z_n - z_m - z)B(z)dz \tag{33}$$

Then we deduce the current vector as: $I = dA^{-1}Z^{-1}\delta E$

We can calculate the input admittance as a function of the number of segments $Y_0 = I_0/V_0$.

III- RESULTS AND DISCUSSION

In this section, we give a quantitative discussion about the responses of the single CNT dipole antenna. For setting some problem parameters, we begin firstly by studying the convergence of the antenna response. As illustrated in Fig.6, the real and imaginary part of conductance converges for a segments number M greater than 50 for the pulse and the triangular basis function. The results are obtained for M=100 segments to ensure the convergence.

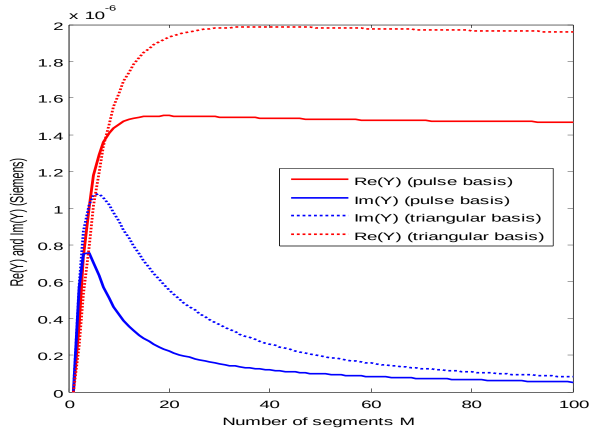


Fig.6: Real and imaginary part of admittance obtained for pulse and basis function.

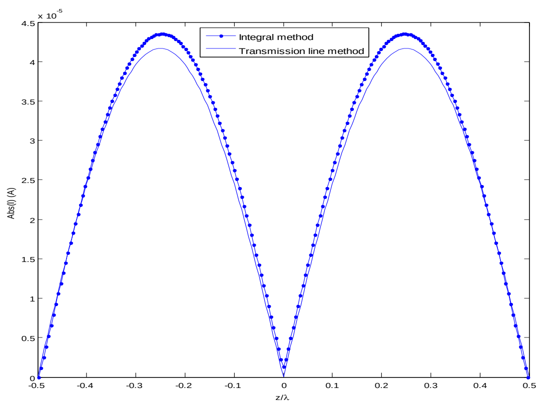


Fig.7: Current distribution of CNT antenna with a radius a=2.71nm and length $l = \lambda_p$.

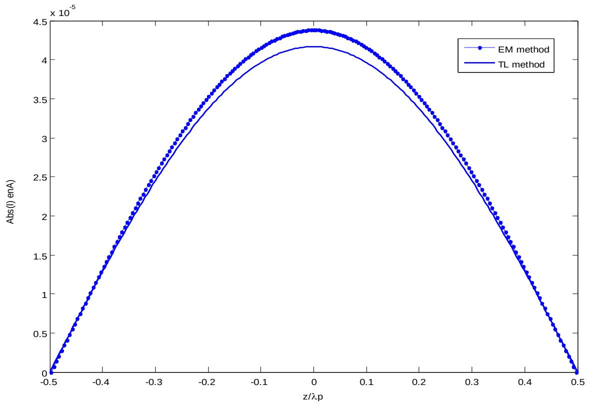


Fig.8: Current distribution of CNT antenna with a radius a=2.71nm and, length l= $\lambda_p/2$.

The Current distribution along the CNT antenna has been calculated for different approaches. Obtained current distribution for an operating frequency equal to 10 GHz and applied gap-slice source of unit voltage, are shown in Fig.7 and Fig.8. In both cases there's small difference in the magnitude, while keeping the same shape of current distribution. This difference is due to the choice of the basis function.

We present the radiation pattern of the CNT antenna and conventional thin wire antenna for different lengths and operating frequency f=10GHz. Obtained results are illustrated in Fig.9.

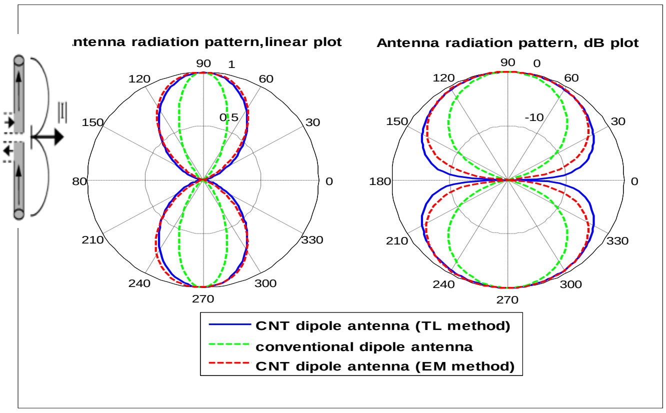


Fig.9: Radiation pattern of CNT antenna with radius *a=2.71nm* and length $l=\lambda_p$.

Note that the conventional thin wire antenna is more directives than the CNT antenna. If the length of the conventional thin wire antenna is of the order of 0.01λ, the two antennas generate the same radiation pattern.

The far-field electrical field is the sum of electric field of each element with length $l=\lambda_p$. This field is cancelled if the number of elements even, because these elements are out of phase. If there are an odd number of elements, all elements are cancelled except the edges dipole element which radiate. This allows to conclude that an antenna of length equal to an odd number of elements of length $l=\lambda_p$ is similar to an antenna of length $l=\lambda_p$.

IV- Conclusion

Fundamental properties of carbon nanotubes antenna have been investigated via transmission line approach and Hallen's integral equation based on a dynamic conductivity. The paper deals with the assessment of a current distribution induced along a straight CNT excited at its center by the gap-slice source of unit voltage using both electromagnetic approach and transmission line model. Results obtained via different approaches are found to be in satisfactory agreement.

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