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Modeling and Sliding Mode Control of a Quadrotor Unmanned Aerial Vehicle

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***Abstract*—In this paper, a detailed mathematical model for a Quadrotor Vertical Take-Off and Landing (VTOL) type of Unmanned Aerial Vehicles (UAVs) is firstly established for the nonlinear attitude and position control. All aerodynamic forces and moments of the studied Quadrotor UAV are described within an inertial frame. The dynamic model is obtained using the Newton-Euler formalism. A nonlinear Sliding Mode Control (SMC) approach is then designed for this vehicle in order to stabilize its vertical flight dynamics. The tracking of an helical desired trajectory is investigated for the SMC-controlled Quad rotorcraft. Demonstrative numerical simulation are carried out in order to demonstrate the effectiveness of the proposed control approach.**

***Index Terms*—VTOL aircraft, Quadrotor UAV, modeling, flight dynamics, sliding mode control, attitude and position stabilization, Lyapunov theory, path tracking.**

I. Introduction

An Unmanned Aerial Vehicle (UAV) refers to a flying machine without an on-board human pilot [1], [2], [3], [4], [5]. These vehicles are being increasingly used in many civil domains, especially for surveillance, environmental researches, security, rescue and traffic monitoring.

Researchers have led to different designs for this type of aircrafts. A Quadrotor UAV is one of the Vertical Take-Off and Landing (VTOL) designs which are proven to have promising flying concepts due to their high maneuverability. The complex mechanical structure of the Quadrotor, its strongly nonlinear and coupled dynamics, its multiple inputs-outputs and the observation difficulty of its states allowed this VTOL aircraft to be a popular topic of research in the field of robotics and nonlinear control theory. So, modeling and control of this kind of nonlinearl systems became increasingly difficult and hard tasks in the practical design and prototyping framework.

Several linear control approaches, such as PID,Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG), have been proposed in the literature and applied for attitude stabilization and/or altitude tracking of Quadrotors [6], [7]. However, these methods can impose limitations on application of Quadrotors for extended flight regions, i.e. aggressive maneuvers, where the system is no longer linear. Moreover, the stability of the closed-loop system can only be achieved for small regions around the equilibrium point, which are extremely hard to compute. In addition, the performances on tracking trajectories of these control laws are not satisfactory enough comparing with other more advanced methods. To overcome this problem, nonlinear control alternatives, such as the feedback linearization [8], SMC [9], [10], [11] and Backstepping [13] approaches are recently used in the VTOL aircrafts control framework. An integral predictive/nonlinear $\mathcal{H}_\infty$ strategy has been also proposed and applied by G.V. Raffo et al. in [12]. In this paper, a nonlinear SMC approach is proposed for the attitude stabilization for a Quadrotor. Roll, pitch and yaw dynamics are separately controlled thanks to Lyapunov-based designed SMC controllers. A nonlinear model of the studied UAV is firstly established using the Newton-Euler formulation.

The remainder of this paper is organized as follows. Section II presents the flight dynamics modeling of the Quadrotor UAV based on the well known Newton-Euler approach. Section III is devoted to design a nonlinear SMC approach for the UAV flight stabilization and path tracking. All numerical simulation results, obtained for modeling and control, are presented and discussed in Section IV. Section V concludes this paper.

II. Modeling of the Quadrotor UAV

Design and analysis of control systems are usually started by carefully considering mathematical models of physical systems. In this section, a complete dynamical model of the studied Quadrotor UAV is established using the Newton-Euler formalism.

A. System description and aerodynamic forces

A Quadrotor is an UAV with four rotors that are controlled independently. The movement of the Quadrotor results from changes in the speed of the rotors. The structure of Quadrotor in this paper is assumed to be rigid and symmetrical. The center of gravity and the body fixed frame origin are coincided. The propellers are rigid and the thrust and drag forces are proportional to the square of propeller's speed. The studied Quadrotor rotorcraft is detailed with their body- and inertial-frames $\boldsymbol{F}_b\left(b, x^b, y^b, z^b\right)$ and $\boldsymbol{F}_i\left(G, X^G, Y^G, Z^G\right)$ respectively, as shown in Fig. 1.

Let consider the following model partitions naturally into translational and rotational coordinates [1], [3], [4], [5]:

$$\xi = (x, y, z) \in \mathbb{R}^3, \quad \eta = (\phi, \theta, \psi) \in \mathbb{R}^3 \qquad (1)$$

where $\xi = (x, y, z)$ denotes the position vector of the center of mass of the Quadrotor relative to the fixed inertial frame,

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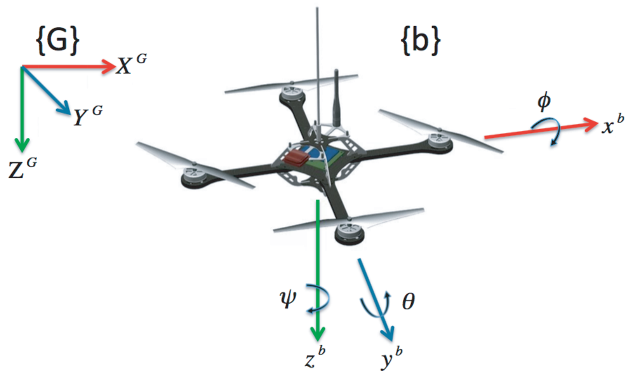

Fig. 1. Mechanical structure of the Quadrotor and related frames.

$\eta = (\phi, \theta, \psi)$ denotes the attitude of the Quadrotor given by the Euler angles ϕ, θ and ψ.

We note that, ϕ is the roll angle around the x-axis, θ is the pitch angle around the y-axis and ψ are the roll angle around the z-axis. All those angles are bounded as follows:

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2} \tag{2}$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \tag{3}$$

$$-\pi < \psi < \pi \tag{4}$$

Each motor M_i (i=1, 2, 3 and 4) of the Quadrotor produces the force which is proportional to the square of the angular speed. Known that the motors are supposedly turning only in a fixed direction, the produced force F_i is always positive. The front and rear motors (M1 and M3) rotate counter-clockwise, while the left and right motors (M2 and M4) rotate clockwise. As given in [1], [5], [2], the gyroscopic effects and the aerodynamic torques tend to cancel in trimmed flight because the mechanical design of the Quadrotor. The total thrust F is the sum of individual thrusts of each motor. Let denote by m the total mass of the Quadrotor and g the acceleration of the gravity.

The orientation of the Quadrotor is given by the rotation matrix $R : \boldsymbol{F}_i \rightarrow \boldsymbol{F}_b$ which depends on the three Euler angles (ϕ, θ, ψ) and defined by the following equation:

$$R(\phi,\theta,\psi) = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\theta & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{5}$$

where $c(.) = \cos(.)$ and $s(.) = \sin(.)$.

During its flight, the Quadrotor is subjected to external forces like the gusts of wind, gravity, viscous friction and others self generated such as the thrust and drag forces. In addition, external torques are provided mainly by the trust of rotors and the drag on the body and propellers. Moments generated by gyroscopic effects of motors are also noted.

The trust force generated by the i_{th} rotor of the Quadrotor is given by:

$$F_i = \frac{1}{2}\rho\Lambda C_T r^2 \omega_i^2 = b\omega_i^2 \tag{6}$$

where ρ is the air density, r and Λ are the radius and the section of the propeller respectively, C_T is the aerodynamic thrust coefficient.

The aerodynamic drag torque, caused by the drag force at the propeller of the i_{th} rotor and opposed the motor torque, is defined as follows:

$$\delta_i = \frac{1}{2}\rho\Lambda C_D r^2 \omega_i^2 = d\omega_i^2 \tag{7}$$

where C_D is the aerodynamic drag coefficient.

The pitch torque is a function of the difference $(F_3 - F_1)$, the roll torque is proportional to the term $(F_4 - F_2)$ and the yaw one is the sum of all reactions torques generated by the four rotors and due to the shaft acceleration and propeller drag. All these pitching, rolling and yawing torques are defined respectively as follows:

$$\tau_\theta = l\,(F_3 - F_1) \tag{8}$$

$$\tau_\phi = l\,(F_4 - F_2) \tag{9}$$

$$\tau_\psi = c\,(F_1 - F_2 + F_3 - F_4) \tag{10}$$

where c is a constant coefficient and l denotes the distance from the center of each rotor to the center of gravity.

Two gyroscopic effects torques, due to the motion of the propellers and the Quadrotor body, are additively provided. These moments are given respectively by:

$$M_{gp} = \sum_{i=1}^{4} \Omega \wedge \left[\ 0, 0, J_r\,(-1)^{i+1}\,\omega_i\ \right]^T \tag{11}$$

$$M_{gb} = \Omega \wedge J\Omega \tag{12}$$

where Ω is the vector of the angular velocity in the fixed earth frame and $J = diag\,[I_x, I_y, I_z]$ is the inertia matrix of the Quadrotor, I_x, I_y and I_z denote the inertias of the x-axis, y-axis and z-axis of the Quadrotor, respectively, J_r denotes the z-axis inertia of the propellers' rotors.

The Quadrotor is controlled by independently varying the speed of the four rotors. Hence, these control inputs are defined as follows:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ -lb & 0 & lb & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{13}$$

where $b > 0$ and $d > 0$ are two parameters depending on the air density, the geometry and the lift and drag coefficients of the propeller as given in Eq. (6) and Eq. (7), and $\omega_{1,2,3,4}$ are the angular speeds of the four rotors, respectively.

From Eq. (13), it can be observed that the input u_1 denotes the total thrust force on the Quadrotor body around the z-axis, the inputs u_2 and u_3 represent the roll and pitch torques, respectively. The input u_4 represents the yawing torque.

B. Modeling with Newton-Euler formalism

While using the Newton-Euler formalism for modeling, the Newton's laws lead to the following motion equations of the Quadrotor:

$$\begin{cases} m\ddot{\xi} = F_{th} + F_d + F_g \\ J\dot{\Omega} = M - M_{gp} - M_{gb} - M_a \end{cases} \tag{14}$$

where $F_{th} = R(\phi,\theta,\psi)\left[0,0,\sum_{i=1}^{4} F_i\right]^T$ denotes the total thrust force of the four rotors, $F_d = diag(\kappa_1,\kappa_2,\kappa_3)\,\dot{\xi}^T$ is the air drag force which resists to the Quadrotor motion, $F_g = [0,0,mg]^T$ is the gravity force, $M = [\tau_\phi,\tau_\theta,\tau_\psi]^T$ represents the total rolling, pitching and yawing torques, M_{gp} and M_{gb} are the gyroscopic torques and $M_a = diag(\kappa_4,\kappa_5,\kappa_6)\left[\dot{\phi}^2,\dot{\theta}^2,\dot{\psi}^2\right]^T$ is the torque resulting from the aerodynamic frictions.

Substituting the position vector and the forces expressions intoEq. (14), we have the following translational dynamics of the Quadrotor:

$$\begin{cases} \ddot{x} = \frac{1}{m}(c\phi c\psi s\theta + s\phi s\psi)\,u_1 - \frac{\kappa_1}{m}\dot{x} \\ \ddot{y} = \frac{1}{m}(c\phi s\psi s\theta)\,u_1 - \frac{\kappa_2}{m}\dot{y} \\ \ddot{z} = \frac{1}{m}c\phi c\theta u_1 - g - \frac{\kappa_3}{m}\dot{z} \end{cases} \tag{15}$$

From the second part of Eq. (14), and while substituting each moment by its expression, we deduce the following rotational dynamics of the rotorcraft:

$$\begin{cases} \ddot{\phi} = \frac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} - \frac{J_r}{I_x}\bar{\Omega}_r\dot{\theta} - \frac{\kappa_4}{I_x}\dot{\phi}^2 + \frac{1}{I_x}u_2 \\ \ddot{\theta} = \frac{(I_z - I_x)}{I_y}\dot{\phi}\dot{\psi} - \frac{J_r}{I_y}\bar{\Omega}_r\dot{\phi} - \frac{\kappa_5}{I_y}\dot{\theta}^2 + \frac{1}{I_y}u_3 \\ \ddot{\psi} = \frac{(I_x - I_y)}{I_z}\dot{\theta}\dot{\phi} - \frac{\kappa_6}{I_z}\dot{\psi}^2 + \frac{1}{I_z}u_4 \end{cases} \tag{16}$$

where $\kappa_{1,2,\ldots,6}$ are the drag coefficients and positive constant, $\bar{\Omega}_r = \omega_1 - \omega_2 + \omega_3 - \omega_4$ is the overall residual rotor angular velocity.

Taking $X = \left(\phi,\dot{\phi},\theta,\dot{\theta},\psi,\dot{\psi},x,\dot{x},y,\dot{y},z,\dot{z}\right)^T$ as state vector, the following state-space representation of the studied Quadrotor is obtained as follows:

$$\dot{X} = f(X,u) = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1x_4x_6 + a_3\bar{\Omega}_r x_4 + a_2x_2^2 + b_1u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4x_2x_6 + a_6\bar{\Omega}_r x_2 + a_5x_4^2 + b_2u_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7x_2x_4 + a_8x_6^2 + b_3u_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = a_9x_8 + \frac{1}{m}(c\phi c\psi s\theta + s\phi s\psi)\,u_1 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = a_{10}x_{10} + \frac{1}{m}(c\phi s\theta s\psi - s\phi c\psi)\,u_1 \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = a_{11}x_{12} + \frac{c\phi c\theta}{m}u_1 - g \end{cases} \tag{17}$$

where:

$$a_1 = \frac{I_y - I_z}{I_x};\ a_2 = -\frac{\kappa_4}{I_x};\ a_3 = -\frac{J_r}{I_x};\ a_4 = \frac{(I_z - I_x)}{I_y};$$
$$a_5 = -\frac{\kappa_5}{I_y};\ a_6 = -\frac{J_r}{I_y};\ a_7 = \frac{(I_x - I_y)}{I_z};$$
$$a_8 = -\frac{\kappa_6}{I_z};\ a_9 = -\frac{\kappa_1}{m};\ a_{10} = -\frac{\kappa_2}{m};$$
$$a_{11} = -\frac{\kappa_3}{m};\ b_1 = \frac{1}{I_x};\ b_2 = \frac{1}{I_y};\ b_3 = \frac{1}{I_z}$$

III. Sliding Mode Control of the Quadrotor

A. Basic concepts of SMC

The SMC is a type of Variable Structure Control (VSC). Its basic idea is to attract the system states towards a surface, called sliding surface, suitably chosen and design a stabilizing control law that keeps the system states on such a surface. For the choice of the sliding surface shape, the general form of Eq. (18) was proposed by Stoline and Li in [13]:

$$S(x) = \left(\lambda_x + \frac{d}{dt}\right)^{q-1} e(x) \tag{18}$$

where x denotes the variable control (state), $e(x)$ is the tracking error defined as $e(x) = x - x_d$, λ_x is a positive constant that interprets the dynamics of the surface and q is the relative degree of the sliding mode controller.

Condition, called attractiveness is the condition under which the state trajectory will reach the sliding surface. There are two types of conditions of access to the sliding surface. In this paper, we will use the Lyapunov based approach. It consists to make a positive scalar function, given by Eq. (19) and called Lyapunov candidate function, for the system state variables and then choose the control law that will decrease this function:

$$\dot{V}(x) < 0, \ \text{ with } V(x) > 0 \tag{19}$$

In this case, the Lyapunov function can be chosen as:

$$V(x) = \frac{1}{2}S(x)^2 \tag{20}$$

The derivative of this above function is negative when the following expression is checked:

$$S(x)\dot{S}(x) < 0 \tag{21}$$

The purpose is to force the system state trajectories to reach the sliding surface and stay on it despite the presence of uncertainty. The sliding control law contains two terms as follows:

$$u(t) = u_{eq}(t) + u_D(t) \tag{22}$$

where $u_{eq}(t)$ denotes the equivalent control which is a way to determine the behaviour of the system when an ideal sliding regime is established. it is calculated from the following invariance condition of the surface:

$$\begin{cases} S(x,t) = 0 \\ \dot{S}(x,t) = 0 \end{cases} \tag{23}$$

and $u_D(t)$ is a discontinuous function calculated by checking the condition of the attractiveness. It is useful to compensate the uncertainties of the model and often defined as follows:

$$u_D(t) = -K sign(S(t)) \tag{24}$$

where K is a positive control parameter and $sign(.)$ is the sign operator.

B. SMC controllers design for the Quadrotor

For the attitude control, we use the rotational motion model given by Eq. (16). The translational dynamics model of Eq. (15) is used to design the Quadrotor position controller. Let also consider the state vector given by Eq. (17).

We begin by defining the tracking errors which represent the difference between the set-point and current values of the state:

$$\begin{cases} e_{i+1} = \dot{e}_i \\ e_i = x_i - x_{id}, i = 1, 2, \dots, 11 \end{cases} \tag{25}$$

The sliding surfaces are chosen based on the tracking errors such as:

$$\begin{cases} S_\phi = e_2 + \lambda_1 e_1 \\ S_\theta = e_4 + \lambda_2 e_3 \\ S_\psi = e_6 + \lambda_3 e_5 \\ S_x = e_8 + \lambda_4 e_7 \\ S_y = e_{10} + \lambda_5 e_9 \\ S_z = e_{12} + \lambda_6 e_{11} \end{cases} \tag{26}$$

Let consider for the roll dynamics SMC design the following Lyapunov function:

$$V(S_\phi) = \frac{1}{2} S_\phi^2 \tag{27}$$

While referring to Eq. (19) and Eq. (21), we deduce the expression of the derivative roll surface given as:

$$\dot{S}_\phi = -K_1 sign(S_\phi) \tag{28}$$

By changing $\dot{x}_2$ with its expression and referring to the above equations, the control law u_2 is given by:

$$u_2 = \frac{1}{b_1} \begin{bmatrix} -a_1 x_4 x_6 - a_3 \bar{\Omega}_r x_4 - a_2 x_2^2 + \\ \ddot{x}_{1d} - \lambda_1 \dot{e}_1 - K_1 sign(S_\phi) \end{bmatrix} \tag{29}$$

While following exactly the same steps as the roll controller design, the control inputs u_3 and u_4, responsible of generating the pitch and yaw rotations respectively, are calculated as follows:

$$u_3 = \frac{1}{b_2} \begin{bmatrix} -a_4 x_2 x_6 - a_6 \bar{\Omega}_r x_2 - a_5 x_4^2 + \\ \ddot{x}_{3d} - \lambda_2 \dot{e}_3 - K_2 sign(S_\theta) \end{bmatrix} \tag{30}$$

$$u_4 = \frac{1}{b_3} \begin{bmatrix} -a_7 x_2 x_4 - a_8 x_6^2 + \\ \ddot{x}_{5d} - \lambda_3 \dot{e}_5 - K_3 sign(S_\psi) \end{bmatrix} \tag{31}$$

Using the same method, we deduced the control laws u_1, u_x and u_y for the stabilization of z, x and y positions of the Quadrotor, respectively. These control inputs are computed as follows:

$$u_1 = \frac{m}{c\phi c\theta} \left[-a_{11} x_{12} + \ddot{x}_{11d} - \lambda_6 \dot{e}_{11} - K_6 sign(S_z) + g \right] \tag{32}$$

TABLE I
QUADROTOR MODEL PARAMETERS.

Parameters	Values and units
Lift coefficient b	2.984 e-05$N.s^2/rad^2$
Drag coefficient d	3.30 e-07 $N.s^2/rad^2$
Mass m	0.5 kg
Arm length l	50 cm
Motor inertia J_r	2.8385 e-05 $N.m/rad/s2$
Quadrotor inertia J	$diag(0.005, 0.005, 0.010)$
aerodynamic friction coeffs. $\kappa_{1,2,3}$	0.3729
translational drag coeffs. $\kappa_{4,5,6}$	5.56 e-04
acceleartion of the gravity g	9.81 m/s^2

$$u_x = \frac{m}{u_1} \left[-a_9 x_8 + \ddot{x}_{7d} - \lambda_4 \dot{e}_7 - K_4 sign(S_x) \right] \tag{33}$$

$$u_y = \frac{m}{u_1} \left[-a_9 x_{10} + \ddot{x}_{9d} - \lambda_5 \dot{e}_9 - K_5 sign(S_y) \right] \tag{34}$$

IV. Simulation Results And Discussion

In this section, the proposed SMC approach for the Quadrotor attitude stabilization is implemented in order to verify his validity and efficiency. For the simulation, we use the physical parameters of Table I . The initial position and angle values are set as $[0, 0, 0]$ m and $[0, 0, 0]$ rad.

Even though the reference angle were changed in every moment, the proposed control scheme managed to effectively hold the quadrotor's attitude in finite-time, as shown in Fig. 2 and Fig. 3 for the attitude dynamics control, and in Fig. 4 and Fig. 5 for the position dynamics tracking. In Fig. 6, we present the helical trajectory tracking of the Quadrotor. It is shown that even though the quadrotor's attitude and position are affected by the abruptly changed reference angles, the designed SMC controllers are able to drive all these state variables back to the new reference angle and position within seconds. Moreover, the aerodynamic forces and moments are taken into account in the controllers design. Those demonstrate the robustness of the proposed control strategy and its effectiveness.

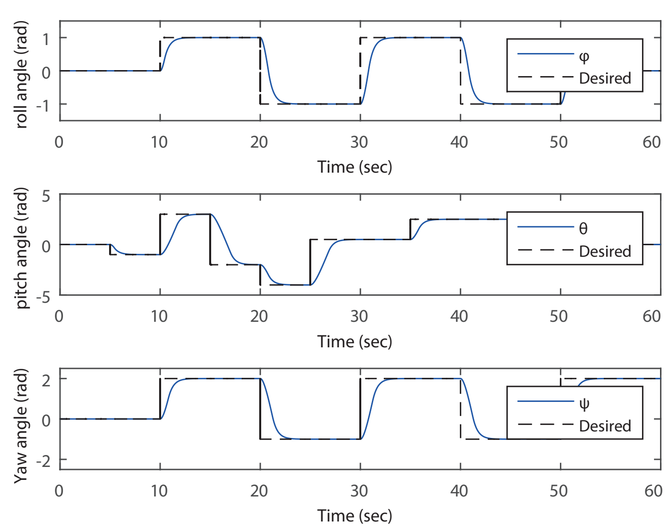

Fig. 2. SMC- based results for the attitude tracking of the Quadrotor.

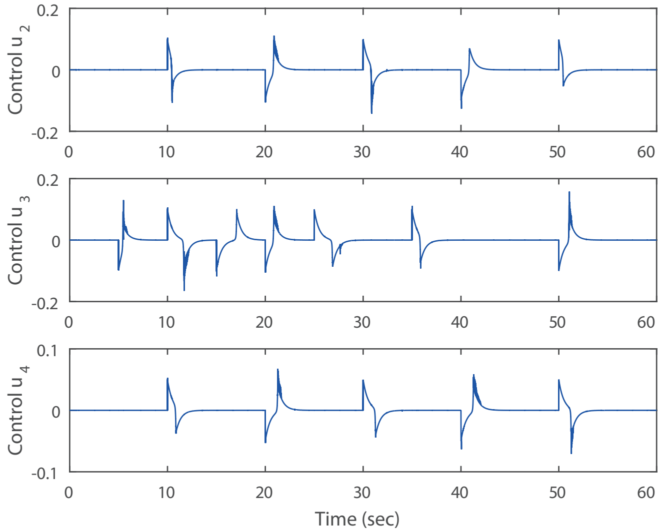

Fig. 3. Control inputs for the attitude dynamics tracking.

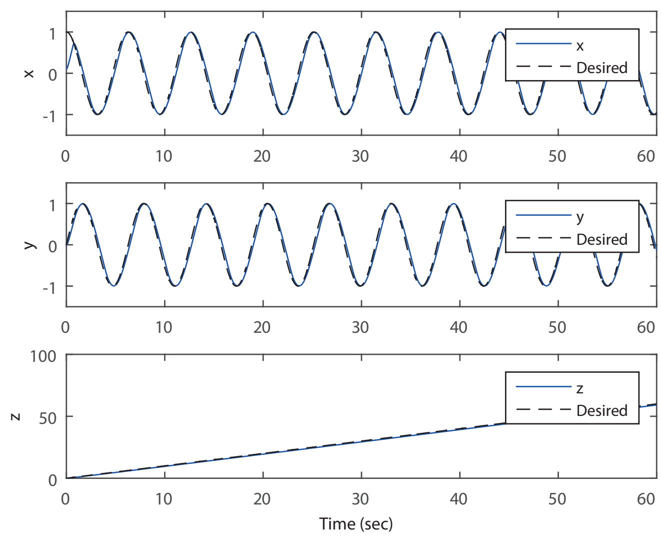

Fig. 4. SMC based results for the position tracking of the Quadrotor.

V. CONCLUSION

In this paper, we deal with the problem of the stabilization and tracking of a Quadrotor vehicle using a nonlinear sliding mode control approach. Firstly, the development of a dynamic nonlinear model of the Quadrotor, taking into account the different physics phenomena and aerodynamic forces and moments, is presented thanks to the Newton-Euler formalism. Sliding mode controllers are then designed based on the Lyapunov theory to stabilize and track the Quadrotor attitude and position. Several simulations results are carried out in order to show the effectiveness of the proposed modeling and nonlinear control methodology. Forthcoming works deal with the tuning and the optimization of all SMC parameters with metaheuristics-based approaches. In addition, the Hardware-In-the-Loop (HIL) co-simulation of the designed SMC approach will be also investigated.

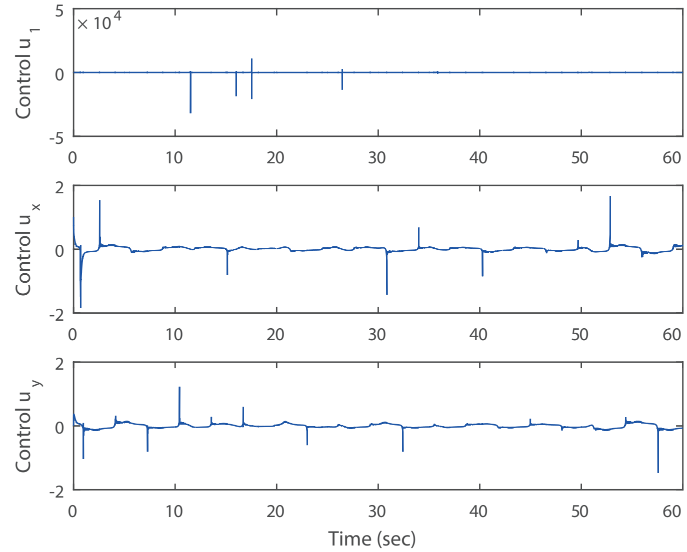

Fig. 5. Control inputs for the position dynamics tracking.

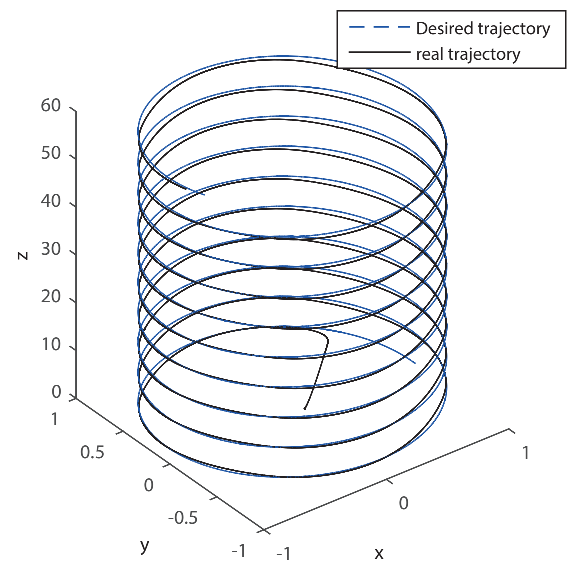

Fig. 6. SMC- based helical path tracking of the Quadrotor.

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