

A New Scheme for Timing Offset Estimation in Fast Varying Channel

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Abstract—This paper deals with the symbol timing issue of an OFDM system in fast varying channel. Symbol timing offset (STO) estimation is a major task in OFDM. Most of existing methods for estimating STO used cyclic prefix or training sequences. In this paper, we consider a new system for STO estimation using constant amplitude zero auto-correlation (CAZAC) sequences as pilot sequences in conjunction with fractional Fourier transform (FRFT). This method gives good results in terms of MSE in comparison with other known techniques and it is important for fast varying channel. MATLAB Monte-Carlo simulations are used to evaluate the performance of the proposed estimator.

Index Terms—OFDM, STO, FRFT, CAZAC sequences, MSE, SNR.

I. INTRODUCTION

Multicarrier modulations are increasingly used in various telecommunication systems such as in Digital Audio Broadcasting (DAB), Digital Video Broadcasting Terrestrial (DVB-T), digital broadband communications, Long Term Evolution (LTE), WiMAX,... The OFDM system carries the message data on orthogonal subcarriers for parallel transmission, combating the distortion caused by the frequency-selective channel or equivalently, the inter-symbol-interference in the multi-path fading channel.

IFFT and FFT are the basic functions needed for the modulation and demodulation at the transmitter and receiver of OFDM systems, respectively. In order to take the N-point FFT in the receiver, we need the exact samples of the transmitted signal for the OFDM symbol duration. In other words, a symbol-timing synchronization must be performed to detect the starting point of each OFDM symbol, which facilitates obtaining the exact samples. Therefore, STO must be estimated by the receiver. Estimated STO is then compensated with called timing synchronization.

Timing synchronization is one of the major task of the receiver in OFDM system. Imperfect synchronization destroys the orthogonality of sub-carriers and degrades the performance of OFDM system. Timing synchronization includes symbol timing offset estimation and correction. Many techniques are used literature to compensate this STO, using cyclic prefix

or training sequences as preamble. These methods will be described later in this paper.

In this paper, We implement an OFDM Transmitter with CAZAC sequences as pilot sequences and Fractional Fourier Transform. In reception, STO estimator is implemented.

The remainder of this paper is organized as follows. In section 2, we introduce OFDM signal and the effect of STO. Then, we present Fractional Fourier Transform in section 3. Thereafter, Section 4 shows the proposed method. Finally, the last one shows the performance of this technique in terms of MSE.

II. OFDM SIGNAL AND STO

OFDM signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. Let us define N symbols in OFDM as $\{X_n, n = 0, 1, \dots, N - 1\}$. The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by :

$$x_l(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_l(k) e^{j2\pi \Delta f k t}; 0 \leq t < NT \quad (1)$$

Where $j = \sqrt{-1}$, Δf is the subcarrier spacing, l le l^{th} OFDM symbol and NT denotes the useful data block period. In OFDM systems the subcarriers are assumed to be mutually orthogonal (i.e., $\Delta f = \frac{1}{NT}$).

In order to demodulate an OFDM symbol correctly at the receiver using N-point DFT (Discrete Fourier transform), it is very much required to take exact samples of transmitted OFDM symbol. The correct starting point of DFT window is required to preserve the orthogonality between sub-carriers. There is lot of advantages of OFDM system over single carrier system however all these advantages can be useful only when the orthogonality among sub-carriers is maintained. If one DFT window takes sample of two different OFDM symbol then it will generate Inter-carrier interference (ICI) and Inter-Symbol interference (ISI). Table I shows the effect of timing offset in the received signal in time and frequency domain the effects of channel and noise are neglected for simplicity of exposition.

	Received signal	Effect of STO δ on the received signal
Time-domain signal	$y(n)$	$x(n + \delta)$
Frequency-domain signal	$Y(k)$	$e^{j\frac{2\pi k\delta}{N}} X(k)$

TABLE I
THE EFFECT OF STO ON THE RECEIVED SIGNAL, [7].

Note that the STO of δ in the time domain incurs the phase offset of $2\pi kd/N$ in the frequency domain, which is proportional to the subcarrier index k as well as the STO δ . Four possible cases may occur, Figure 1.

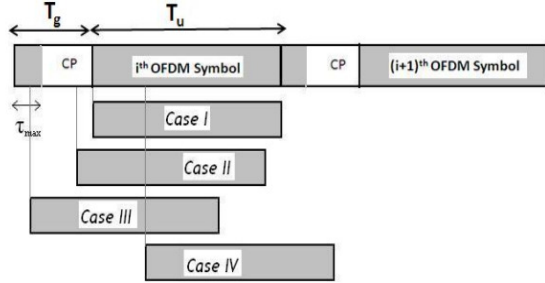


Fig. 1. Four different cases of OFDM symbol starting point subject to STO

Case I : First consider the case when there is no timing error (i.e. timing offset $\delta = 0$). This is the case when the estimated starting point of OFDM symbol coincides with the exact timing, preserving the orthogonality among sub-carriers, therefore the OFDM symbol can be perfectly recovered without any type of interference.

Case II : This is the case when the estimated starting point of OFDM symbol is before the exact point, yet after the end of the (lagged) channel response to the previous OFDM symbol. In this case, the i^{th} symbol is not overlapped with the previous $(i-1)^{th}$ OFDM symbol, that is, without incurring any ISI by the previous symbol in this case. Consider the received signal in the frequency domain by taking the FFT of the time-domain received samples $\{x_l(n + \delta)\}_{n=0}^{N-1}$, given as:

$$\begin{aligned}
 Y_l(k) &= \frac{1}{N} \sum_{n=0}^{N-1} x_l(n + \delta) e^{-j2\pi nk/N} \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} \left\{ \sum_{p=0}^{N-1} X_l(p) e^{j2\pi(n+\delta)p/N} \right\} e^{-j2\pi nk/N} \quad (2) \\
 &= X_l(k) e^{j2\pi k\delta/N}
 \end{aligned}$$

The expression in Equation 2 implies that the orthogonality among subcarrier frequency components can be completely preserved. However, there exists a phase offset that is proportional to the STO δ and sub-carrier index k , forcing the signal constellation to be rotated around the origin.

Case III: This is the case when the starting point of the OFDM symbol is estimated to exist prior to the end of the (lagged) channel response to the previous OFDM symbol,

and thus, the symbol timing is too early to avoid the ISI. In this case, the orthogonality among sub-carrier components is destroyed by the ISI (from the previous symbol) and furthermore, ICI occurs.

Case IV: This is the case when the starting point of the OFDM symbol is estimated just after the exact point. In this case, the samples for current FFT operation interval consists of a part of the current OFDM symbol $x_l(n)$ and a part of next OFDM symbol $x_{l+1}(n)$.

As shown, an STO may cause not only phase distortion but also ISI in OFDM systems. In order to warrant its performance, therefore, the starting point of OFDM symbols must be accurately determined by estimating the STO with a synchronization technique at the receiver. In general, STO estimation can be implemented either in the time or frequency domain.

Many techniques in the literature are implemented using whether cyclic prefix or training sequence. As an example of estimating STO using cyclic prefix is done by Tourtier, P.J., Monnier, R., and Lopez, P. [1].

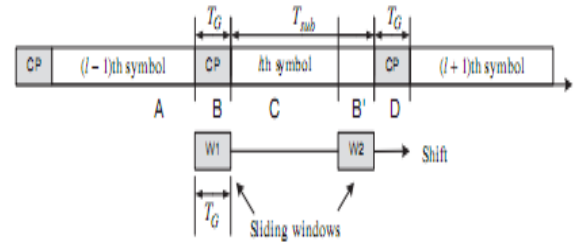


Fig. 2. STO estimation technique using double sliding windows.

The estimated STO is done by :

$$\hat{\delta} = \arg \min_n \left\{ \sum_{i=\delta}^{N_G-1+\delta} |y_l(n+i) - y_l(n+N+i)| \right\} \quad (3)$$

Another technique used in [5]. It consists on minimizing the squared difference between a N_G sample block (seized in window W1) and conjugate of another N_G sample block (seized in window W2).

$$\hat{\delta} = \arg \min_n \left\{ \sum_{i=\delta}^{N_G-1+\delta} (|y_l(n+i)| - |y_l^*(n+N+i)|)^2 \right\} \quad (4)$$

Estimation techniques using training sequences are presented in Figure 3 :

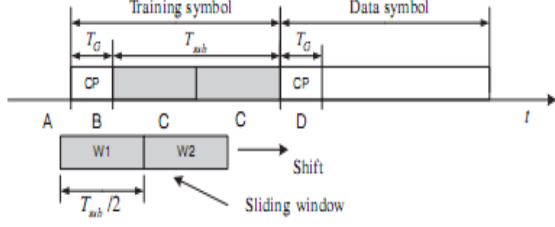


Fig. 3. STO estimation using the repetitive training symbol, (period = $T_{sub}/2$).

In this case, STO is estimated as, [3] and [4] :

$$\hat{\delta} = \arg \min_n \left\{ \sum_{i=\delta}^{N_G-1+\delta} (|y_i(n+i) - y_i^*(n+N/2+i)|)^2 \right\} \quad (5)$$

In this paper, STO is estimated in frequency domain. As implied in equation 2, the received signal subject to STO suffers from a phase rotation, Figure 4:

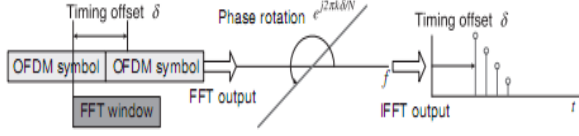


Fig. 4. STO estimation in frequency domain.

STO is estimated as :

$$\hat{\delta} = \arg \max_n (y_i^X(n)) \quad (6)$$

Where $y_i^X(n)$ is defined as :

$$y_i^X(n) = IFFT\{Y_i(k)e^{j2\pi\delta k/N}X_i^*(k)\} \quad (7)$$

In this paper, we are using CAZAC sequence and fractional Fourier transform in stead of FFT in figure 4.

III. FRACTIONAL FOURIER TRANSFORM (FRFT)

The fractional Fourier Transform is a generalization of the Fourier Transform. The FRFT of a signal $s(t)$ is defined as follows : $F_\alpha(s) = S_\alpha(u) = \int_{-\infty}^{+\infty} s(t)K_\alpha(t,u)dt$. where p is a real number known as FRFT order, $\alpha = p\frac{\pi}{2}$ is the angle of FRFT, and $K_\alpha(t,u)$ is the kernel of FRFT:

$$= \begin{cases} \sqrt{\frac{1-j\cot(\alpha)}{2\pi}} \exp(j\frac{t^2+u^2}{2}\cot(\alpha) - jut\csc(\alpha)) & \alpha \neq n\pi \\ \delta(t-u) & \alpha = 2n\pi \\ \delta(t+u) & \alpha + \pi = 2n\pi \end{cases}$$

The FRFT can be considered as a projection of the signal on an axis which forms an angle α with the time axis: a rotation in the time-frequency plane that generalizes FFT. The FRFT gives great satisfactions in many signal processing applications

such optical communications, signal filtering and also beam forming for fading channels, [8].

Multicarrier modulation that uses traditional Fourier Transform attempts a frequency windowing of bandwidth. The effect of the time-invariant channel distortions can be compensated for by subchannel-by-subchannel basis single-tap frequency-domain equalizers. Consequently, the overall traditional multicarrier system can be seen as an optimal Fourier-domain filter. However, if the channel is time-varying, the traditional multicarrier system loses optimality since optimal recovery operator is generally time-variant. This means that it cannot be implemented in the conventional Fourier domain and is exactly the reason that motivates the use of an FRFT-based technique.

IV. PROPOSED METHOD

In this section, the proposed method is presented. A conventional OFDM system is used but Fractional Fourier Transform FRFT block is used in stead of classical FFT. We use CAZAC sequences as pilot sequences. The pilot sequences are inserted in combo-type model, Fig 5. The Timing Offset estimation is done in frequency domain. Estimated STO is obtained by multiplying the received pilot sequences (with STO) by the conjugated pilot sequence. CAZAC sequences used in this paper are defined as :

$$X_p((k-1)*N_{ps}+1) = e^{j*\pi*(k-1)^2/N_p} \quad \text{for } k = 1 \dots N_p \quad (8)$$

Where N_{ps} and N_p are pilot spacing and number of pilot sequences respectively in OFDM symbol.

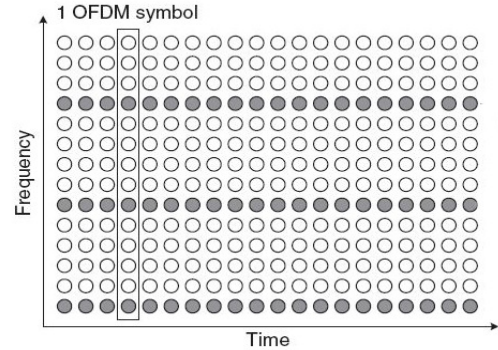


Fig. 5. Comb-Type pilot arrangement

The proposed receiver schema is given by figure 6:

V. SIMULATION RESULTS AND DISCUSSIONS

To evaluate the performance of the proposed methods, computer simulations are established. Parameters of this simulation are listed in Table II.

Figure 7 shows the Mean Square Error (MSE) of the symbol timing Offset (STO) of the OFDM system using Fractional Fourier Transform and CAZAC sequences. This figure shows the superiority of the proposed system in terms of

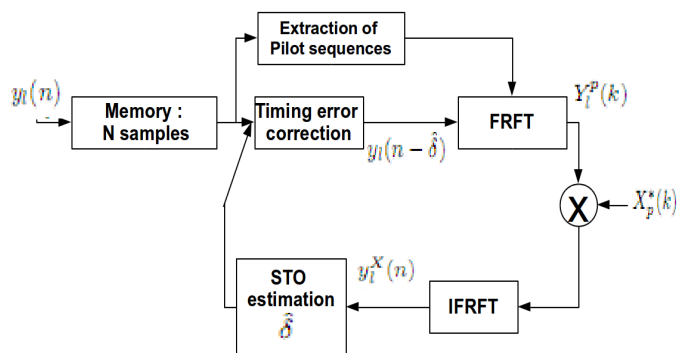


Fig. 6. STO synchronization scheme using pilot tones.

Modulation	16 QAM
Number of sub-carrier	128
Cyclic Prefix Length (N_g)	32
Pilot sequences	CAZAC sequences
Number of Bits per Symbol	4
Pilot Spacing (N_{ps})	4
Normalized Timing offset (STO)	-3
Signal to Noise Ratio (SNR)	0:30

TABLE II
THE PARAMETERS FOR SIMULATION.

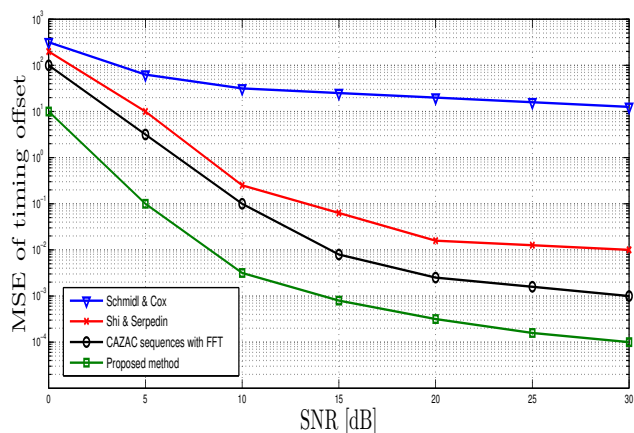


Fig. 7. MSE of STO estimation for proposed method compared to known method : Schmidl and Cox, Shi and Serpedin.

MSE by comparison to competitive systems such as Schmidl and Cox [3], Shi and Serpedin [5] methods. Simulation also proves the effect of fractional Fourier transform in comparison with classical Fourier transform. The proposed method shows attractive results compared to GR method proposed by G. Ren, Y. Chang, and H. Zhang, [6].

This method is useful in fast varying channel that varies from OFDM symbol to another and does not decrease much

the useful throughput in comparison with the methods using training sequences.

Although the proposed method has good efficiency in term of MSE, it has a greater complexity in comparison with other STO estimation methods.

VI. CONCLUSION

This paper proposes a new symbol timing offset (STO) estimation that uses CAZAC sequences as pilot sequences in conjunction with Fractional Fourier Transform. The main design criterion of this method is to exploit the well-known efficiency of both CAZAC sequences and FRFT in reducing MSE of STO of the designed system. The system we designed shows attractive performance and stands useful for mobile fast varying channels.

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