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Super Twisting Control for Attitude Tracking using Quaternion

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***Abstract*—A nonlinear robust control based on super Twisting algorithm for the attitude stabilisation and tracking of quadrotor is developed in this paper. Quaternion based representation is used to obtain the nonlinear model and avoid singularities problems. The stability of the Super Twisting control based on quaternion can be proved through Lyapunov function candidate. The experimental results show the robustness and finite time convergence of the control in presence of parameters uncertainties and external disturbances .**

Index - Super Twisting algorithm, quaternion representation, sliding variable, robustness, quadrotor plateform.

I Introduction

Unmanned aerial vehicles (UAVs) are a part of the future. They are being used more often for military and civilian purposes such as traffic monitoring, patrolling for forest fires, surveillance, and rescue, in which risks to pilots are often high.
The most used class of the UAV is the quadrotor, it has an evident advantage comparing to the other classes for various applications because of its vertical landing/take-off capability, payload, great maneuverability and easy to manufacture.
For this, the quadrotor becomes an interesting area of research. Various methods are developed to control the quadrotor position. We find the linear algorithms which deal with the system locally around its equilibrium point as LQR [1] and PID [2].
Since the quadrotor is an underactuated system, that is it has six degrees of freedom to be controlled and only four inputs, and highly coupled model, nonlinear controls were taken into consideration to deal with this latter. Serval approaches are developed in this direction such as backstepping [3], sliding mode control [4], Super twisting control [5] and adaptive backstepping [6]. The used representation in most researches is based on Euler angle which leads to control loss when the quadrotor passes by the position ($\phi = \pi/2$ or $\theta = \pi/2$ or $\psi = \pi$). We call this problem singularities. Quaternion based representation is the solution to avoid this problem, there are researches that have take attention to this representation where they developed various control such as chen and Lo [7] , Taybi [8] and Wu Shunan [9].

This paper presents the second order sliding mode control Super Twisting algorithm (STWA) for quaternion-based spacecraft attitude tracking and stabilisation. In our work we define firstly in section II, the quadrotor model using the quaternion representation associated with the fixed and mobile frames, where we get the kinematic and dynamic equations.
in section III, we compute the control law by defining a sliding variable and using the STWA.
Finally, we present experimental results that show the validation of the theoretical conclusions about stability, robustness and finite time convergence.

II Mathematical Model of Quadrotor

The quadrotor consists of a rigid cross airframe with four individual rotors as seen in Fig1. The front and rear rotors, numbered 1 and 3, rotate counterclockwise (positive about the z-axis), while the left and right rotors, numbered 2 and 4, rotate in a clockwise direction. Vertical motion is achieved by increasing or decreasing the speed of each rotor by the same proportion. The roll motion is controlled by increasing the thrust of rotor 2 (4) and decreasing the thrust of rotor 4 (2) to obtain a positive (negative) roll to the right (left). The pitch motion is achieved similarly by differential speed between rotors 1 and 3. The yaw motion of the quadrotor is achieved by adjusting the average thrust of the clockwise and counterclockwise rotating rotors. When a yaw motion in the positive direction is desired for example, the rotor pair 1 and 3 increase by the same proportion, while the rotor pair 2 and 4 decrease by the same proportion. This will maintain the same overall aircraft thrust without pitching or rolling the aircraft.

To get the attitude equations of the UAV there are different ways. In our work we are intersting by the quaternion representation to avoid singularities. The quaternion is given by:

$$Q = q_0 + q_1 i + q_2 j + q_3 k \tag{1}$$

where q_0 is a scalar and $q_1 i + q_2 j + q_3 k$ is a vector.
The quaternion is described by the following propriety :

$$\|Q\| = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2}$$

Let us consider the quadrotor as being a rigid body under external forces applied to its center of mass, the dynamic equation referred to the body coordinates system under the

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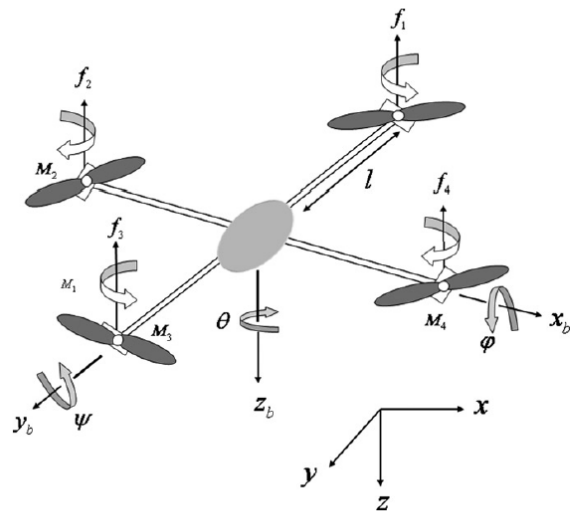

Fig. 1. Quadrotor model

Newton-Euler formulation is :

$$\begin{bmatrix} mI_{3\times3} & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega \times mV \\ \omega \times J\omega \end{bmatrix} = \begin{bmatrix} u_z \\ u \end{bmatrix} \quad (3)$$

Using the kinematic and dynamic attitude equations, quaternion representation are given by :

$$\dot{Q} = \frac{1}{2}S(Q)\omega \quad (4)$$
$$\dot{\omega} = J^{-1}(-[\omega\times]J\omega + u + d) \quad (5)$$

where d is the bounded disturbance.

$$S(Q) = \begin{bmatrix} q_0 I_{3\times3} + [q\times] \\ -q^T \end{bmatrix} \quad (6)$$

$[q\times]$ is a skew matrix defined by :

$$[q\times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \quad (7)$$

$V = [v_x \quad v_y \quad v_z]$ is the translation velocity.
$\omega = [\omega_x \quad \omega_y \quad \omega_z]$ is the angular velocity vector;
$\bar{u} = [u \quad u_z] = [u_1 \quad u_2 \quad u_3 \quad u_z]$ is the control vector, that are the torques and the thrust forces generated by the four DC motors and is given by :

$$u_1 = \tau_\phi = lb(\Omega_2^2 - \Omega_4^2) \quad (8)$$
$$u_2 = \tau_\theta = lb(\Omega_1^2 - \Omega_3^2) \quad (9)$$
$$u_3 = \tau_\psi = \bar{\rho}(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (10)$$
$$u_z = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$

Ω_i are the angular speed of the four rotors respectively.

Let us introduce now the motor dynamique which contains electrical and mechanical equations. This model is composed of the series of a resistor $R[\Omega]$, an inductor $L[H]$ and a voltage generator $e[V]$. The resistor represents the Joule loss due to the current flow into the copper conductor. Its value depends on geometric and material characteristics such as wire resistivity, length and section.
The model of the DC motor is represented in figure bellow :

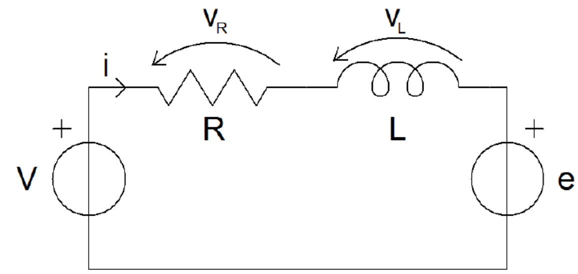

Fig. 2. Motor circuit

The equations describing the motor are given by :

$$v = \frac{di}{dt} + Ri + K_e\Omega \quad (11)$$
$$J_m\dot{\Omega} = C_{em} - C_r \quad (12)$$

The inductor part is neglected because it is small and the electric part is so faster then the mechanical one; so the model will be as :

$$v = Ri + K_e\Omega \quad (13)$$
$$J_m\dot{\Omega} = -\frac{K_M K_e}{R}\Omega + \frac{K_M}{R}v \quad (14)$$

v is the voltage input: the real input of the system, K_M, K_e are mechanic motor constant and electric motor constant respectively and R is the motor resistance.

III CONTROL DESIGN

To design the control law, we use the second order sliding mode algorithm called Super Twisting. The control goal is to get a good performance in term of stabilisation and attitude tracking.
Let consider the following sliding variable :

$$s = e_\omega + \lambda e_q \quad (15)$$

Where $\lambda = [\lambda_{q1} \quad \lambda_{q2} \quad \lambda_{q3}]$ is a constant gain vector.
e_q the quaternion error given by :

$$e_q = q - q_d \quad (16)$$

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q_d is the desired position in the quaternion frame.
e_ω is the angular velocity error.

$$e_\omega = \omega - \omega_d \tag{17}$$

ω_d is the desired angular velocity.

To complete the control design we use two properties related to the quadrotor motion equation that is given as follow [7] :
Property 1 :
The matrix $S(Q)$ has the following properties :

$$\begin{aligned} S(Q)^T S(Q) &= I_{3\times 3} \\ \|S(Q)\| &= 1 \\ \frac{d}{dt}[S(Q)^T \dot{Q}] &= S(Q)^T \ddot{Q} \\ \|\omega\| &= 2\,\|S(Q)\| \end{aligned}$$

Using (3) and the previous properties, the desired angular velocity can be expressed as follow :

$$\omega_d = 2S(Q)^T \dot{Q}_d \tag{18}$$
$$\dot{\omega}_d = 2S(Q)^T \ddot{Q}_d \tag{19}$$

So the dynamic of sliding variable can be written as :

$$\dot{s} = \dot{\omega} - \dot{\omega}_d + \lambda(\dot{q} - \dot{q}_d) \tag{20}$$

The super twisting controller is given as follow :

$$u = J(J^{-1}[\omega\times]J\omega - J^{-1}d + \dot{\omega}_d - \frac{1}{2}\lambda S(Q)\omega + \lambda\dot{q}_d + z) \tag{21}$$

Where z is the super twisting correcting term expressed by:

$$z = -k_1\,|s|^{\frac{1}{2}}\,sign(s) - k_2\int_0^t sign(s(\tau))d\tau + \nu \tag{22}$$

Where $k_1 = [k_{11} \quad k_{12} \quad k_{13}]$ and $k_2 = [k_{21} \quad k_{22} \quad k_{23}]$ are positive gains.
The closed loop sliding variable by introducing the control (21) is rewritten as :

$$\dot{s} = -k_1\,|s|^{\frac{1}{2}}\,sign(s) + \mu \tag{23}$$
$$\dot{\mu} = -k_2 sign(s) + \varrho \tag{24}$$

$\varrho = \dot{\nu}$ represents the dynamic of parameter uncertainties and external disturbances.

IV EXPEREMENTAL RESULTS

In this work we use a Qanser platform to validate our control. It contains a fixed quadrotor connected with a control card (Q8 usb) and powered by an amplifier as shown in Fig.3. The used sensors are encoders to mesure the angle positions and the actuators are DC motors (Motor-Pittman 9234S004). Experimental testing has been performed using Q8 usb card in combination with Matlab-Simulink- that allows us a real time visualisation and interaction. The four DC motors are powered by Quanser linear voltage amplifier driven by PWM signals.
We are carrying out the control signal via Simulink in block

Fig. 3. Quanser quadrotor

diagram format using the physical parameters given in table I and we obtain the following results due to many tests to show the performances of the proposed control.
Since, we are dealing with the attitude stabilisation and tracking of a fixed quadrotor platform, we suppose that the thrust force u_z is constante to compensate gravity force.
The gain values used in the next expirement are described in the table II :

TABLE I
QUANSER PARAMETERS

parameter	description	value	Unit
m	mass	2.85	Kg
l	distance between Pivot to each Motor	0.1969	m
b	thrust factor	2.98e-6	N/V
$\bar{\rho}$	drag factor	1.14e-7	
J_x	Roll inertia ϕ	0.0552	kgm^2
J_y	Pitch inertia θ	0.0552	kgm^2
J_z	Yaw inertia ψ	0.1104	kgm^2

TABLE II
CONTROL GAINS

parameter	value	parameter	value	parameter	value
λ_{q_1}	2	k_{11}	200	k_{21}	40
λ_{q_2}	2	k_{12}	200	k_{22}	50
λ_{q_1}	1.5	k_{13}	150	k_{23}	20

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IV-A Attitude Stabilisation

Figure (4) represents the quaternion response. This test has been realised by giving the quadrotor initial conditions that are equivalent to a Euler angles values as it is shown in figure (5). The initial conditions are ($q_0 \approx 0.96$, $q_1 \approx 0.2$, $q_2 \approx 0.2$,$q_3 \approx -0.1$) corresponding to ($\theta = 18°$; $\phi = 18°$; $\psi = -9°$). The obtainted results show the power of the control in term of stabilisation.

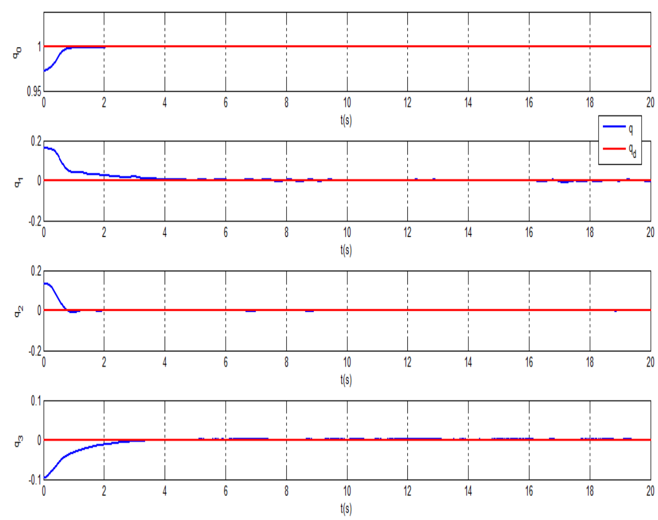

Fig. 4. Quaternion trajectories for stabilisation test

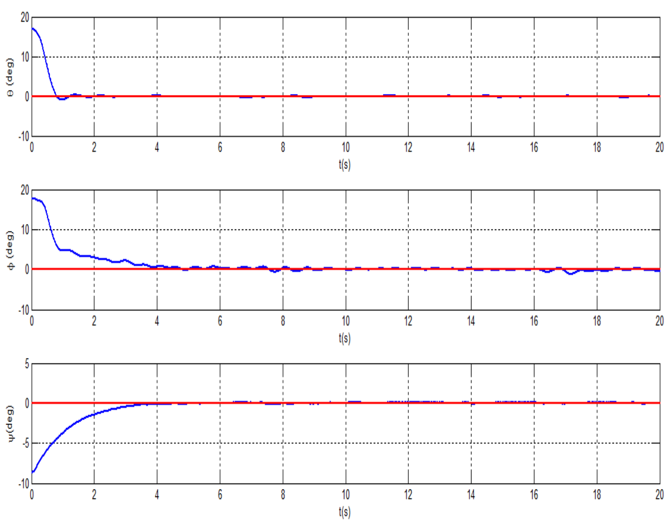

Fig. 5. Corresponding Euler angles trajectories for stabilisation test

IV-B Attitude Tracking

In this experiment, we give the machine a desired trajectories with sinusoidal form. Figures (6,7) show the high performance of the control in term of tracking. The small oscillations in the response trajectories are due to the sensor noise and drift.

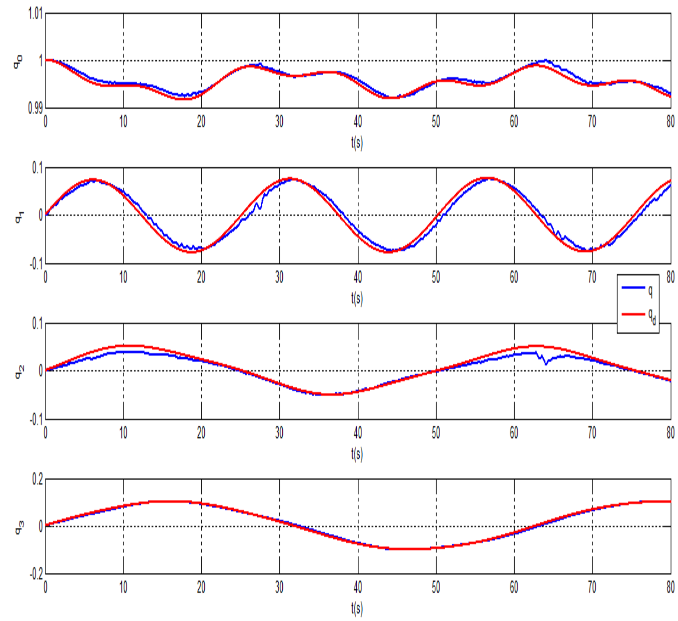

Fig. 6. Quaternion trajectories for tracking test

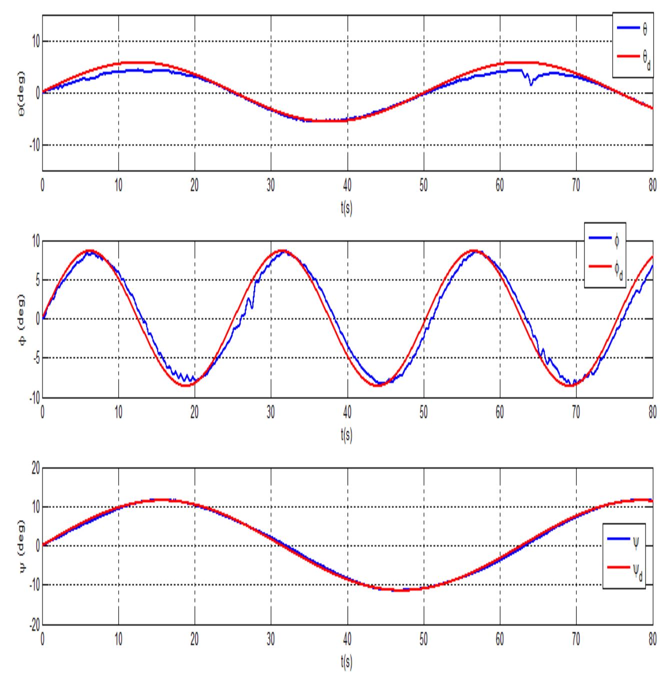

Fig. 7. Corresponding Euler angles trajectories for tracking test

IV-C Robustness Test

Figures (8,9) present the quaternion and the corresponding Euler angles tracking responses. Where we applied a manual

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external force considered as external perturbations.
The disturbances was applied at $t = 15s$ on the θ orientation, at $t = 30s$ on the ϕ axes and at $t = 45s$ in both orientations. The effectiffness of the super twisting appears in this experience. In fact, we clearly see that there is a good rejection of the external diturbances and a quick reaction, which means that the control signal intervene so that the output follows the desired trajectory.

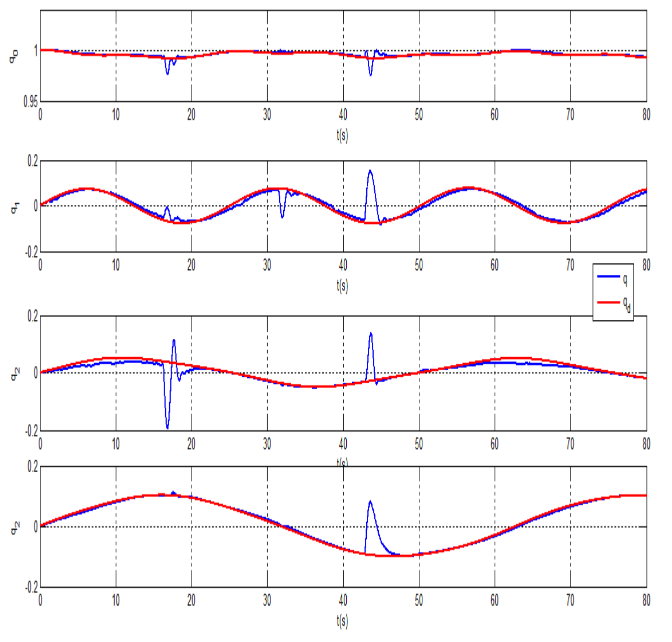

Fig. 8. Quaternion trajectories for robustness test

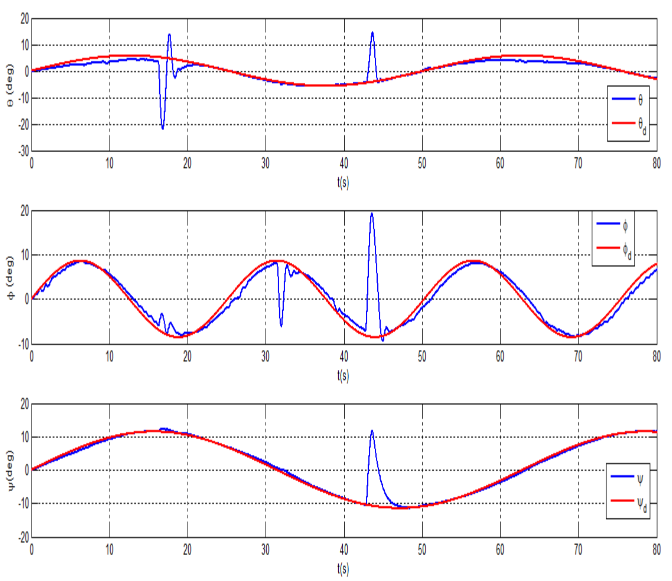

Fig. 9. Corresponding Euler angles trajectories for robustness test

V Conclusion

In this paper, a finite time control algorithm has been proposed for the attitude stabilisation and tracking of a quadrotor system. The controller has been designed using second order sliding mode approach based on quaternion in order to avoid singularities and to ensure robustness with respect to parameters uncertainties and external disturbances. Experimental results obtained via quanser platform show the ability of the controller to ensure the stabilisation and tracking even in the presence of external disturbances and model uncertainties .

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