

An Empirical Mode Decomposition Signal Denoising Method Based on Novel Thresholding

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Abstract— Empirical Mode Decomposition (EMD) was used for signal denoising in wide range of applications such as biomedical signals and acoustic signals. In this paper, a novel signal denoising method named (EMD-New) based on EMD and the novel Thresholding Function is proposed. EMD was applied to the noisy signal to decompose adaptively the signal into intrinsic mode functions (IMFs). Then, all the noisy IMFs were thresholded to suppress noise in the signal and improve the output signal to noise ration (SNR). To verify the effectiveness of the method, simulated data and real ECG signal were used, and compared to traditional methods EMD-Soft and EMD-Hard thresholding. The performances were evaluated using output SNR in dB, and Mean Square Error (MSE). The results illustrated that the proposed approach is significantly more effective in removing the noise components from the noisy signal.

Keywords— ECG signal, EMD, Soft-thresholding, Hard-thresholding, Novel -thresholding.

I. INTRODUCTION

The Empirical Mode Decomposition (EMD) [1] is an algorithm for analyzing the nonlinear and non-stationary signals such as ECG signals. The aim of the EMD method is to adaptively represent the signals as sums of zero-mean oscillating components, called the intrinsic mode functions (IMFs) using a sifting process. The signal reconstruction process is achieved by total sum of the IMFs and the residual. In recent years, EMD has been widely used in signal denoising [2]-[10]. The denoising method can be based on the signal estimation using all the IMFs previously thresholded as in wavelet analysis [2],[3]. The noise components of a noisy signal are centered on the first IMFs (high-frequency IMFs) and the useful information of the signal is often concentrated on the last IMFs (low-frequency IMFs)[4]. Thereby, the denoising method can also be based on the partial construction of the signal using only the last relevant IMFs and the EMD residual [4], [5]. Many EMD-Based denoising methods using thresholding were proposed in [6]-[9]. EMD thresholding techniques introduced by [6]-[9] are also used to suppress noise in the signal and improve the output signal to noise ration SNR. Indeed, it was shown that the direct application of wavelet thresholding to IMFs can lead to very bad results for the continuity of the reconstructed signal. The main factors affecting the quality of denoising are threshold and selection of the suitable threshold function. The hard threshold function does not change the local properties of the signal, but it can lead to some fluctuation in the reconstruction of the original signal. The hard threshold function leads to a loss of some

high frequency coefficients above the threshold. In order to overcome the drawbacks of the classical threshold functions. S. Liu et al [11] proposed a novel threshold function. In this study we propose the use of EMD combined with new threshold function [11] in a method denoted as EMD-New for denoising to obtain higher SNR. The proposed method was tested on real ECG signal using the MIT-BIH database [12] and simulated signals (Doppler and Bumps) corrupted by white Gaussian noise and compared to the EMD-Based signal denoising methods using soft, hard thresholding [13]. Simulation results illustrate the effects of the proposed methods. The results were quantitatively evaluated using SNR and mean MSE. We can achieve better denoising performance.

The paper is organized as follows. Section II, introduces the EMD algorithm. Section III, describes the EMD-Soft, EMD hard, and EMD-New thresholding. The simulation results are illustrated in section IV. Finally, section V presents the conclusion.

II. EMD ALGORITHM

EMD is an adaptive method to decompose a signal $x(t)$ into a series of IMFs. The IMFs must satisfy the following two conditions:

- (1) The number of maximum must equal the number of zeros or differ at most by one.
- (2) In each period, it is necessary that the signal average is zero.

The EMD algorithm (the sifting process of extracting IMFs) consists of the following steps [1]:

1. Find local maxima and minima in $x(t)$ to construct the upper and lower envelopes respectively using cubic spline interpolation.
2. Calculate the mean envelope, $m(t)$ by averaging the upper and lower envelopes.
3. Calculate the temporary local oscillation
$$h(t) = x(t) - m(t).$$
4. Calculate the average of $h(t)$ if average $h(t)$ is close to zero, then $h(t)$ is considered as the first IMF, named $c_i(t)$, otherwise, repeat steps (1)–(3) while using $h(t)$ for $x(t)$.
5. Calculate the residue $r(t) = x(t) - c_i(t)$.
6. Repeat steps from (1) to (5) using $r(t)$ for $x(t)$ to obtain the next IMF and residue.

The decomposition process stops when the residue $r(t)$ becomes a monotonic function or a constant that no longer satisfies the conditions of an IMF.

$$x(t) = \sum_{i=1}^N c_i(t) + r_N(t) \quad (1)$$

The sifting process is continued until the last residual is either a monotonic function or a constant.

III. EMD BASED DENOISING

A. EMD Soft thresholding and EMD Hard thresholding

Having a noisy signal $y(t)$ given by:

$$y(t) = x(t) + \eta(t) \quad (2)$$

Where $x(t)$ is the noiseless signal and $\eta(t)$ is independent noise of finite amplitude. In EMD-Soft thresholding method, the noisy signal $y(t)$ is first decomposed into noisy IMFs $c\eta_i(t)$. These noisy IMFs can be thresholded by soft function in order to obtain an estimation of the IMFs $\hat{c}_i(t)$ of the noiseless signal. In this work the universal threshold is used proposed in [14] and it identified as follows:

$$\tau_i = C\sqrt{E_i 2\ln(n)} \quad (3)$$

Where C is a constant, n is the length of the signal and E_i is given by “(4),”.

$$\hat{E}_k = \frac{E_1^2}{0.719} 2.01^{-k}, \quad k=2,3,4,\dots,n \quad (4)$$

Where E_1^2 is the energy of the first IMF defined by:

$$E_1^2 = \left(\frac{\text{median}(|c\eta_1(t)|)}{0.6745} \right)^2 \quad (5)$$

Flandrin et al. [4] proposed a method for estimating the energy of noisy IMFs from a theoretical model and IMFs energies of the test signal. The principle of this method is summarized in the following steps:

1. Apply the EMD process on the noisy signal $y(t)$ to extract the noisy IMFs.
2. Calculate the IMFs energies.
3. Estimate the noise only IMF energies E_k by the by “(4),” [4]:

In EMD Soft thresholding, the Soft thresholding [13] function is defined as follows:

$$\hat{c}_i(t) = \begin{cases} c\eta_i(t) - \tau_i & \text{if } c\eta_i(t) \geq \tau_i \\ 0 & \text{if } |c\eta_i(t)| < \tau_i \\ c\eta_i(t) + \tau_i & \text{if } c\eta_i(t) \leq -\tau_i \end{cases} \quad (6)$$

In EMD Hard thresholding, the thresholding [13] function is defined as follows:

$$\hat{c}_i(t) = \begin{cases} c\eta_i(t) & \text{if } |c\eta_i(t)| > \tau_i \\ 0 & \text{if } |c\eta_i(t)| \leq \tau_i \end{cases} \quad (7)$$

Finally, the denoising signal $\hat{x}(t)$ can be obtained as follows:

$$\hat{x}(t) = \sum_{i=1}^N \hat{c}_i(t) + r_N(t) \quad (8)$$

B. EMD-New Denoising Technique

The new technique suggest decomposition of noisy signal into noisy IMFs using EMD process, and then these noisy IMFs are thresholded using Novel thresholding function [11]. The signal reconstruction is implemented by summing all the IMFs. The *EMD-New* algorithm consists of the following steps:

1. Apply the EMD process on the noisy signal $y(t)$ to extract the noisy IMFs.
2. Calculate the IMFs energies.
3. Estimate the noise only IMF energies E_k by “(4).”

The following Novel thresholding function [11].is given by .

$$\hat{c}_i(t) = \begin{cases} \text{Sign}(c\eta_i(t)) * \sqrt{c\eta_i^2(t) - \tau_i^2} & \text{if } |c\eta_i(t)| \geq \tau_i \\ 0 & \text{if } |c\eta_i(t)| < \tau_i \end{cases} \quad (9)$$

4. Reconstruct the signal using “(8),”.

IV. SIMULATION RESULTS

In order to assess our proposed denoising algorithm, we have performed synthetic signals (Doppler and Bumps) shown in Figure 1, the signals are of identical size $n = 2048$. An additive white Gaussian noise is used Figure 2. The method was also tested on real ECG signal using the MIT-BIH database [12]. For simulated signals, the *SNR* before denoising was maintained at 15 dB. The *SNR* before denoising of the real ECG signal was 20 dB. Each noisy signal was decomposed into IMFs using EMD process, and all IMFs are thresholded by the soft, hard, and novel thresholding functions. The threshold was calculated by multiplication of universal threshold with the constant C “(3),” . The value of the constant C depending of the type of signal. In our work the value of C was set to 0.5. “Fig.3,” shows the plots of the *SNR* after denoising values against the number of samples 1024, 2048, and 4096 of applying EMD-New and EMD-Soft for Doppler and Bumps signals .It is clear that the EMD-New achieve the high performance level. A comparative study with soft and hard thresholding methods considered in this work is presented in Tables I. Clearly, the novel EMD thresholding function gives the best estimates in terms of *SNR* and *MSE* for all test signals. Therefore, the proposed method EMD-New outperforms totally the conventional EMD-Soft and EMD-Hard thresholding methods. Fig. 4 shows the denoising results of applying EMD-Soft and EMD-New to simulated signals. Fig. 5 displays the denoising results of real ECG signal using EMD-Soft and EMD-New. Therefore, we conclude that our algorithm is, in general, able to remove noise from signals and it improves the results obtained by EMD hard thresholding and EMD soft thresholding.

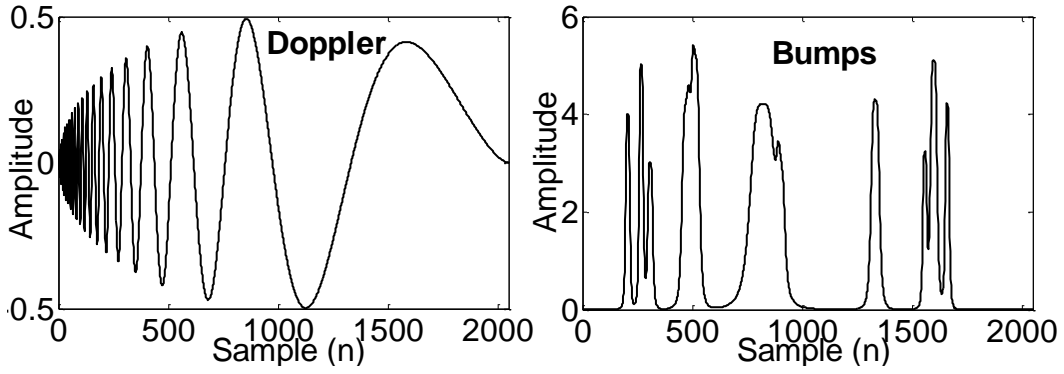


Fig. 1 Test signals with $n=2048$.

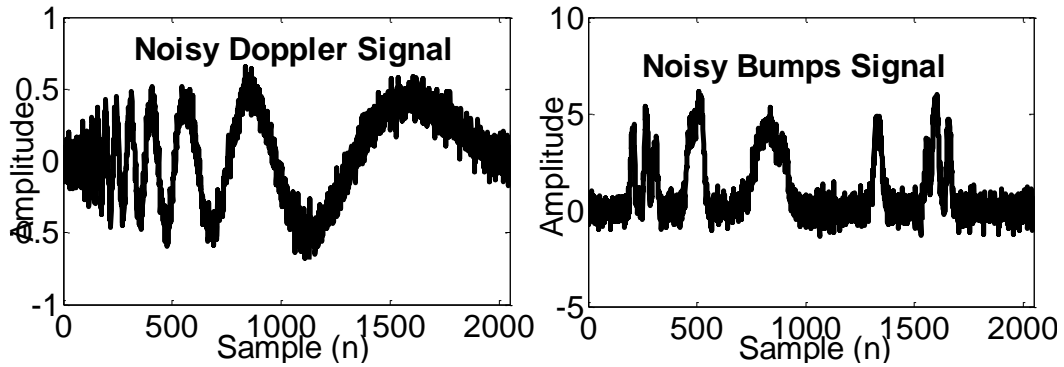


Fig. 2 Noisy test signals.

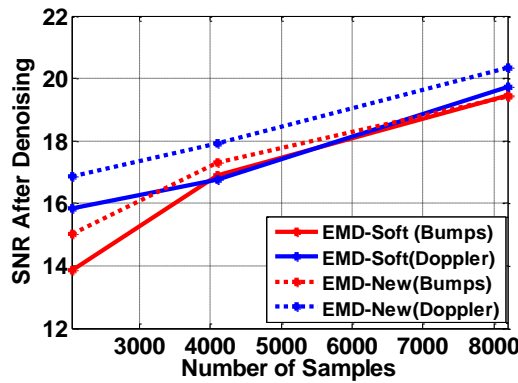


Fig. 3 Performance evaluation of EMD-New and EMD-Soft denoising methods.

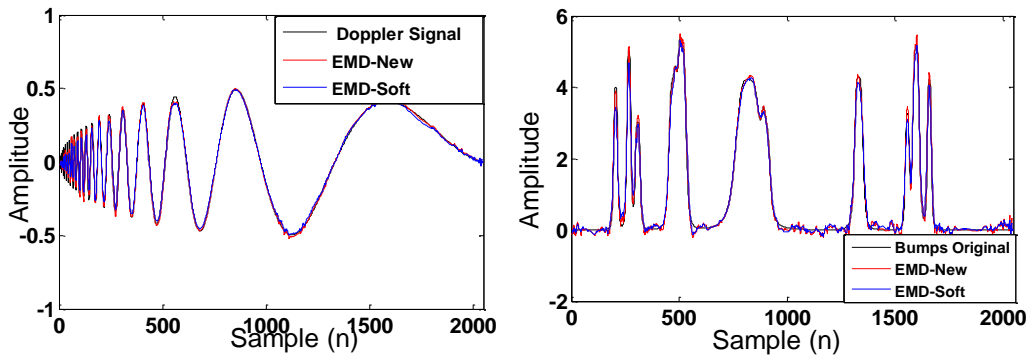


Fig. 4 Denoising results in $SNR = 15dB$ of test signals corrupted by Gaussian noise.

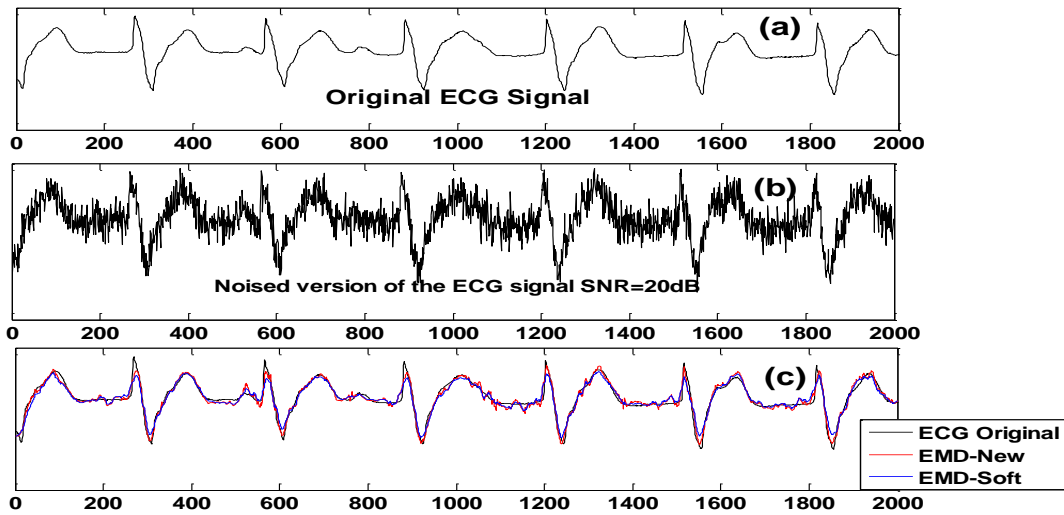


Fig. 5 Denoising results in SNR (20dB) of real ECG signal.

TABLE I
 COMPARISONS OF SNR (AFTER DENOISING) AND MSE VALUES AT $SNR = 5\text{ dB}$

METHODS	SNR (dB) (AFTER DENOISING)				
	BUMPS		DOPPLER		ECG SNR = 20dB
	SNR	MSE	SNR	MSE	SNR
EMD-SOFT	13.8488	0.1335	15.8444	0.0022	26.2561
EMD-HARD	12.7898	0.1704	13.2650	0.0040	27.2952
EMD-NEW	13,9078	0,0926	16.8633	0.0017	27,9064

V. CONCLUSION

In this paper, we proposed a novel signal denoising technique based on empirical mode decomposition and new thresholding function to suppress noise in the signal and improve the output signal to noise ratio SNR . The new proposed technique have been tested on real ECG signal using the MIT-BIH database, and noisy simulated signals (Doppler and Bumps) corrupted by white Gaussian noise. The simulation results are in favour of the new method when compared with EMD-Soft and EMD-hard methods. The performances are evaluated in terms of SNR in dB , and Mean Square Error (MSE). We have demonstrated that the new approach is effective for removing noise.

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