Conférence Internationale en Automatique & Traitement de Signal (ATS-2018)
Proceedings of Engineering and Technology – PET
Vol.35pp.58-63

Free-surface oscillations control of high speed manipulated liquids

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***Abstract*— In this paper, we first propose an equivalent mechanical model to estimate the liquid oscillations in a partially filled container in an industrial packaging process. The purpose of this study is to control the height of the liquid inside the container during and after a high speed manipulation. No measurements are available in real-time, the levels are measured on a testbed via a camera. Several treatments are applied to the pictures to extract the heights of liquid. In order to satisfy various control specifications such as manipulation-time, constraints of liquid height and motor acceleration, a predictive controller is designed to determine the position reference to apply to the motor. The resulting command is used in open-loop into the process.**

I. INTRODUCTION

Sloshing is a common problem with low viscosity batch filling machines in the packaging industry [1]-[2]. In this paper, we present an improved control system for rotary Doypack[1] filling machine. The operation of the machine involves moving a bag through different positioned stations arranged in a circular pattern. The bag is first opened, filled with liquid and finally sealed. The transfer of the bag from the filling station to the sealing station is critical, as the bag is partially filled with fluid. The acceleration of the carrousel carrying the bag causes waves on the surface of the liquid inside the package. The control objective is to run the operation in such a way that the height of the waves never cross a certain level in the bag, in order to guarantee proper sealing of the bag, and to save the machine from contamination and costly machine downtime to clean it.
These operations are performed simultaneously on one, two or three containers, depending on the customer specifications. The rotation of the carousel is the same in every step.
The carousel executes nine rotations of 40 degrees each wich corresponds to a displacement of 0.54 m of the Doypack at each step, reducing the transport time between two stations represents a non-negligible improvement in terms of production capacity and hence lower production cost per filled bag. The machine is powered by a brusless motor located in the axis of the carousel.
The design of the process, client requirements and the existent technology do not allow to mount level sensors in the industrial machine to control liquid oscillations. And since the mouvement is repetitive, such a solution is precluded.
The reference mouvement of the carousel is implemented to the motion control unit in accordance to a "master-slave motion system", a block diagram is shown in fig. 1.

In such a system, one axis is defined as the "Master" axis. The "Virtual master" axis moves every step by the tajectory $y_{Master} = x_{Master}$ from the position $(x_{Master}, y_{Master}) = (0, 0)$ to $(x_{Master}, y_{Master}) = (1, 1)$ (position values are normalized).
In the other hand the "Slave" axis is the actual carousel motor.
The motion controller controls the position of the slave axis as a fuction of the master axis the same from $(x_{Slave}, y_{Slave}) = (0, 0)$ to $(x_{Slave}, y_{Slave}) = (1, 1)$,

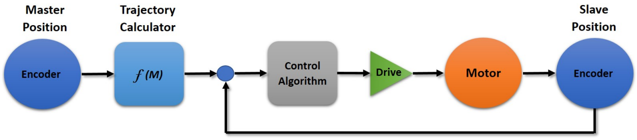

Fig. 1. Master-Slave block diagram

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[1] The Doypack is a semi rigid sealed pouch that stands upright used for almost avery product, it was invented by Louis Doyen in 1962, then president of Thimonnier SAS

ISSN 2356-5608

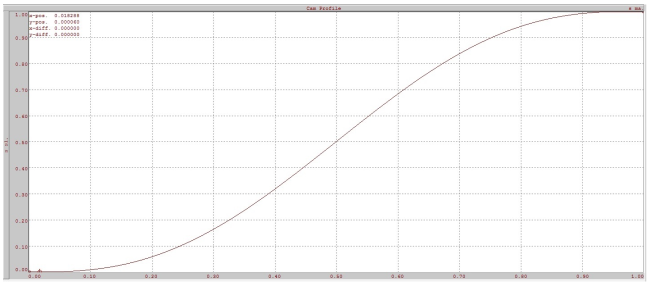

Fig. 2. Slave profile position setting in BR Automation Studio

the fig. 2 shows the profile reference applied to the slave as a function of the master position.

Our objective is to find an open-loop reference position command that minimizes the mouvement time and respects maximum level boundary below the sealing zone.
It is clearly a rest-to-rest motion control problem wich eliminates multiple mode residual oscillations.

II. EXPERIMENTAL APPARTUS

*1) **Mechanical setup:*** In order to perform all the needed experiments, a testbed (see fig. 3) was developped, it is composed of a burshless motor actuating an arm that has the same dimension as the carousel radius $\Phi = 0.76m$, and a pair of non motorized claws to mount the bag. A gear reducer is coupled to the motor to increase the amount of torque it produces, the ratio of reduction is $70 : 1$. The Doypack has a capacity of $V_d = 1L$, a lenght of $L = 0.25m$ and a width $l = 0.18m$.

*2) **Level measurement:*** A vision-based solution was selected over infrared nor ultrasonic technologies since these two are not appropriate for our application. Infrared sensors uses an IR laser transmitter and an IR camera to measure the distance to the surface. If the surface is very reflective, which is our case, the sensor does not work properly. Ultrasonic sensors do not work very well when the dynamics of the oscillations is high. A *Garmin Ultra 30* sport camera is used to measure the level of the liquid, it can perform 300 fps. The camera records the side view of the bag (see fig. 4), an algorithm is used to isolate the liquid from pictures to determine the level of the liquid in the left side of the bag. The fluid is painted with liquid food coloring to make it recognizable in the signal processing and not modify viscosity. In order to minimize signal error, the camera is placed the closest possible to the Doypack while having all the bag in the field of view (FoV). Sports Cameras are equiped with lens that distort images, the output is pictures with a wide FoV, commonly called "Fish-eye".
Such deformation causes non-linear distortion of pixels [17] and consequently wrong fluid level values. A simple method to estimate radial and tangential distortion coefficients is presented in [17], these parameters are then used to undistort the images (see fig. 5).

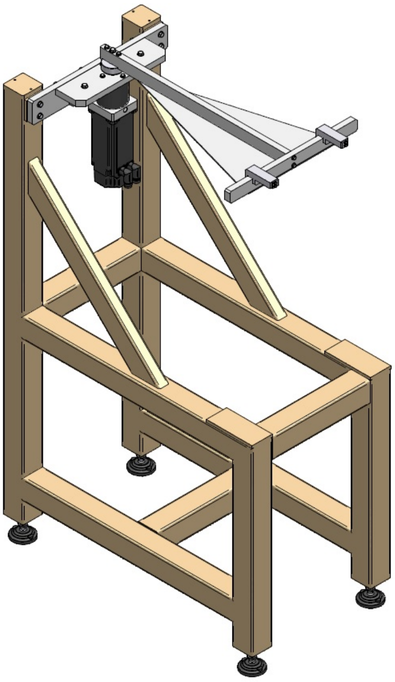

Fig. 3. Testbed 3d mechanical model

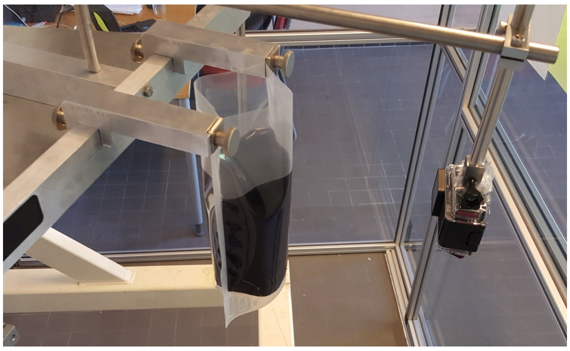

Fig. 4. Level measurement setup

III. MODELING

Several studies have been conducted in modeling of the oscillatory behaviour of liquids in partially filled moving containers in different fields of application [1-8]. An overview of different approaches to model liquid behaviour is presented in [7], but only valid for small amplitude free-surface oscillations. A linear model of the slosh with four states is presented in

Fig. 5. Original image (left) and undistroted image (right). Note the shape red lines corresponding to top and bottom edges of the back white screen, before and after the undistortion

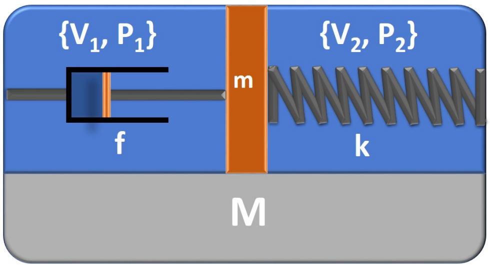

Fig. 6. Equivalent mechanical model of slosh

[1]-[2]. Equivalent mechanical models are used to simulate liquid oscillation since its first introduction in litterature by NASA [9], and prefered for their accuracy and control simplicity. Many of these studies has used pendulums or springs in order to replicate the behaviour of liquids in moving containers [1-10]. Computational Fluid Dynamics (CFD) [11]-[12] cover all numerical methods describing fluids flow behaviour. These methods give good simulation of fluid dynamics under external excitation, however, using CFD models for control is too time-consuming and difficult. CFD models are based on the Navier-Stokes equations [11]-[12].

In this paper, we consider an equivalent mechanical model based on a hydraulic oscillating piston as shown in fig. 6.

The oscillating mass m represents the upper moving part of the liquid on the container, its trajectory simulates the level height in the back side of the Doypack. M is the main mass of the liquid, its position is the position of the Doypack. Using Euler-Lagrage approach, the model dynamic equations are non-linear and have the form :

$$M\ddot{q} + F(q, \dot{q}) = U \tag{1}$$

Where,

$$M = \begin{bmatrix} M+m & m \\ m & m \end{bmatrix} \tag{2}$$

$$F = \begin{bmatrix} 0 \\ kx_3 + fx_4 - \frac{2nRTx_3}{L0^2 - x_3^2} \end{bmatrix} \tag{3}$$

$$U = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4}$$

$$\begin{aligned} \dot{x}(t) &= \mathbf{f}(t, x(t), u(t)) \\ y(t) &= \mathbf{g}(t, x(t)) \end{aligned} \tag{5}$$

Where,

$$\begin{cases} \dot{x_1}(t) = x_2(t) \\ \dot{x_2}(t) = \frac{k-2nRT}{M(L_0^2 - x_3^2(t))} x_3(t) + \frac{f}{M} x_4(t) + \frac{u(t)}{M} \\ \dot{x_3}(t) = x_4(t) \\ \dot{x_4}(t) = \frac{(2nRT-k)(m+M)}{(L_0^2 - x_3^2(t))(m.M)} x_3(t) + \frac{f(m-M)}{m.M} x_4(t) + \frac{u(t)}{M} \end{cases} \tag{6}$$

Also,

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T \tag{7}$$

Where the state variables,

— x_1 : Container position (m)
— x_2 : Container velocity $(m.s^{-1})$
— x_3 : Back level Lvl_B (m)
— x_4 : Time derivative of Lvl_B $(m.s^{-1})$

Parameters are shown in Table I.

IV. CONTROL OF SLOSH

A. Model Predictive Control

Problem formulation :

The problem that we consider consists to solve a

TABLE I

MODEL PARAMETERS

g	Acceleration of gravity $(m.s^{-2})$
M	Main mass (Kg)
m	second mass (Kg)
P_i	Pressure of the chamber i (Pa)
V_i	Volume of the chamber i (m^3)
n	amout of substance of the gas (m)
R	Gas constant (m)
T	Absolute temprature of the gas (K)
f	Friction coefficient $(Kg.s^{-1})$
k	Spring stifness $(N.m^{-1})$

planning trajectory problem. More precisely, we will calculate an open control input which permits to tract the back level x_3 to a desired level output reference $r(t)$. To do so, we solve the problem by using an Model Predictive Control (MPC) algorithm.
In order to simplify the optimization algorithm, we proceed by a linearization of the model around the stady state 0, and we discretize the model to obtain a linear discrete-time model.

$$x(t_{k+1}) = Ax(t_k) + Bu(t_k) \tag{8}$$

MPC is a widely used advanced control technique for dynamic systems under process constraints, known also as moving horizon control.
MPC operates in a receding finite horizon of time N in which it uses an internal dynamic model to determine the optimal input that yields the best predicted system output at every sampling time k.
At each step, current states and forecasts are used by the controller to predict the future behaviour of the system, and current states are used as initial condition of the system to solve a constrained optimal control iteratively.
The solution determines the controls to be used until the next iteration, by minimizing an objective function J_1.
J_1 is a quadratic scalar cost function for output reference tracking, it is defined as follows :

$$\begin{aligned} J_1 = \min_{u(t_k),x(t_k)} \sum_{i=0}^{N-1} & \|r(t_k) - y_1(t_k)\|_Q^2 \\ & + \|u(t_k) - u(t_{k-1})\|_R^2 \end{aligned} \tag{9}$$

s.t.

$$x(t_{k+1}) = Ax(t_k) + Bu(t_k) \qquad \forall k \in \mathbb{N}_0^{N-1} \tag{10}$$
$$x(t_{k+i}) \in \mathcal{X} \qquad \forall k \in \mathbb{N}_0^{N-1} \tag{11}$$
$$u(t_k) \in \mathcal{U} \qquad \forall k \in \mathbb{N}_0^{N-1} \tag{12}$$

Here, Q and R are positive-semi-definite weight matrices, they are chosen in a way to give more importance to certain state or input, and :

- N : Prediction horizon.
- $\mathcal{X}$,$\mathcal{U}$: state and control convex constraints sets.
- $y_1(t_k)$: Predicted level values at kth prediction step.
- $r(t_k)$: Reference values at kth prediction step.
- $u(t_k)$: Input values at kth prediction step.

B. *Reference Tracking*

The Control Variable (CV) is the bag acceleration u, the Manipulated Variables (MVs) is the backward Level of the liquid $y_1(t)$ and the bag position $y_2(t)$.
Many studies was conducted to control and reduce de residual slosh of free-surface liquids or pendulums under lateral excitation [1]-[2]-[3]-[4]-[5]-[6]-[9]-[10]. It was shown in every one of them that the response of the oscillating system resembles to a sinusoidal signal. The half sine waves correspond to the positive and negative accelerations of the container or the cart.
The reference signal is a three degree polynomial $r(t)$ where,

$$r(t) = a(t^3 - 2t^2 + t) \tag{13}$$

clearly we have,

$$\begin{cases} r(0) = 0 \\ r(1) = 0 \\ \dot{r}(1) = 0 \end{cases} \tag{14}$$

This reference permits to start the process at its equilibrium position and to drive it at $t = T_f = 1$ to $x_3(T_f) = 0$. a is the magnitude of the signal that has to be judiciously chosen, depending on the position of the sealing area that has not to be reached by the liquid. A reference bag position can not be set since the optimal problem goal is to find the acceleration profile that minimizes the reference tracking error, it is clearly shown in eq. (6) that the container position depends of the input acceleration and coupled with the liquid level. The final bag position $y_2(T_f)$ is derived from the resulting acceleration profile u, and since the vector u is not known a-priori, $y_2(T_f)$ will be different from the position set-point y_{2d}.
A second optimization problem J_2 is solved to reach the desired position y_{2d}, n is the number of iterations needed to reach it.

$$J_2 = \min_{n \in \mathbb{N}} \sum_{i=n}^{N-1} \|y_2(t_k) - y_{2d}(t_k)\|^2 \tag{15}$$

s.t.

$$n \geqslant \quad n_{min} \tag{16}$$
$$n \leqslant \quad N \tag{17}$$

n_{min} is chosen sufficiently large to prevent too nervous behavior when y_{2d} is reached and guarantee the solvability of the problem.
Thus, the optimization algorithm is formulated in a

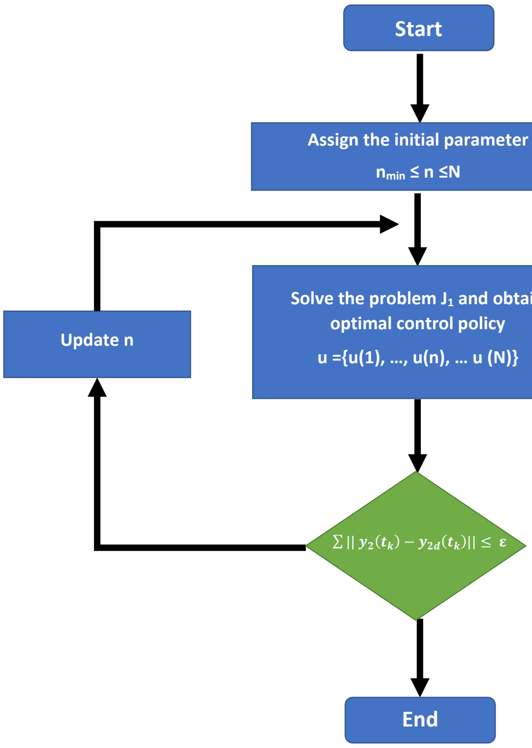

Fig. 7. Optimization algorithm

two-level form. In each iteration, J_2 is optimized, the number of iterations n is then updated and a level reference r is injected to J_1 for trajectory tracking, the optimization stops when the the optimum is reached under the constraints (8), (9) and (10). The algorithm stops when both problems are solved (fig.7).

V. EXPERIMENTAL RESULTS

In this section we present the experimental results from the measurements conducted on the testbed, classic acceleration input profiles in industry and the resulting time-optimal input design are compared.

We have presented in Section II.2 an example of a displacement profile. In such applications, smooth motion profiles are preferred.

In fig. 8, the resulting acceleration profile from the MPC controller is compared to the original acceleration profile used in the past to move the carrousel in $0.75s$. The resulting outputs of previous acceleration profiles are shown in fig. 9 and fig. 10, where, the level reference trajectory (shown in fig .11) was set in order to have a settling time of $0.75s$ and a maximum amplitude of $50mm$ of level height.

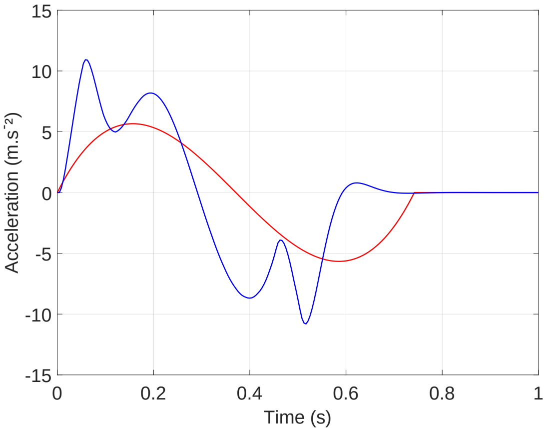

Fig. 8. Acceleration reference inputs, Original acceleration profile (red) and the calculated acceleration profile (blue)

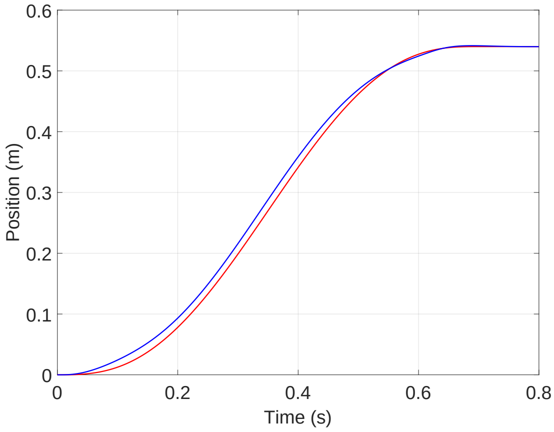

Fig. 9. Position outputs, using the original acceleration profile (red) and the calculated acceleration profile (blue)

Fig .10 shows the behavior of the liquid inside the bag during the two displacement. As a result, slosh control was realized. Compared with the classic control, the proposed control reduces the settling time considering the same transfer time.

VI. CONCLUSIONS

In this paper, we have solved the stady-to-stady liquid level problem in a moving open container from rest to rest. An open control acceleration input was calculated in order to follow a level reference trajectory using a predictive control approach, combined with a second optimization problem for final position set-point.

The resulting control input shows a significant improvement in terms of residual oscillations for the same

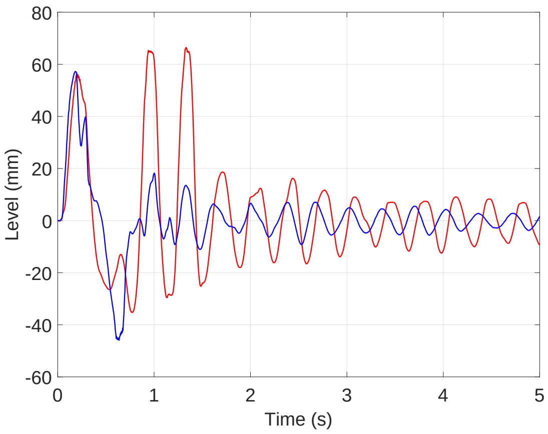

Fig. 10. Measured level outputs, using the original acceleration profile (red) and the calculated acceleration profile (blue)

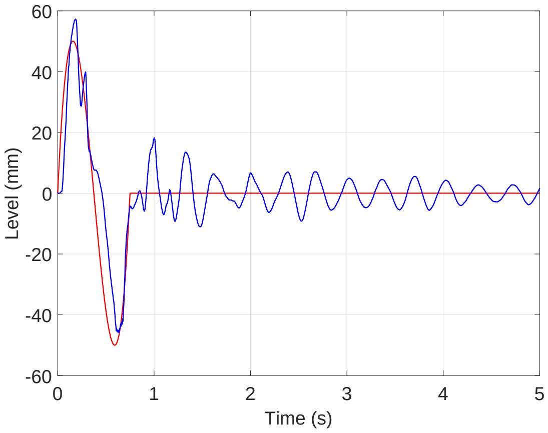

Fig. 11. Level reference (red) versus level measurement (blue)

transfert time 0.75s.

VII. ACKNOWLEDGMENTS

This work was supported by Thimonnier SAS.

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