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Robust Control Scheme for Wireless Networked Control Systems

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***Abstract*— This paper studies the design of an H∞ control approach for uncertain linear discrete-time Wireless Networked Control Systems. Subsequently, the proposed work focuses on the modelling of a dynamic output feedback controller that deal with the different problems (interferences, delays) that can affect the network behaviour in order to maintain the mean square stability and to improve the H∞ disturbance attenuation performance of the studied system. Specifically, the time-varying packet dropouts over imperfect communication channels of the wireless network are introduced as norm-bounded uncertainties and stochastic parameters. The controller state parameters are specified based on an iterative LMI approach. Numerical simulation results, using True Time distributed blocks with communication over a wireless network, are carried out to illustrate the efficiency of the proposed results .**

***Keywords*— Wireless Networked Control Systems – Distributed control – Wireless protocols - Packet dropout – Iterative LMI - Robust control – Uncertainty - Dynamic state feedback controller .**

I. Introduction

The Networked Control Systems (NCSs) are a distributed components (actuators, sensors and controllers) which are interconnected by a shared limited medium communication network.

Obviously, in order to ensure an efficient communication there must be taken into consideration the network effects: congestion, packet loss, transmission delay, interference

Physical wires has been the traditional way of interconnecting these systems. This method is expansive and hard to maintain specially in large networks.

Due to the expansion of the wireless communication, researchers focuses on reaching the network control via wireless networks in order to support emerging media-rich application, to deliver more quality of services to more users in less time and to achieve benefits in terms of reliability, wiring (less cabling), implementation (handling multiple geographically distributed nodes) costs and maintenance (efficient monitoring and less restrictive control action over a distributed computation).

Moreover, the WNCS are considered as a very flexible NCS since, first they could be easily implemented even on resource constrained or low-power node and second, their design could be extended by adding new subsystems.

Further, Wireless NCS are challenging researchers because transmission between neighbour nodes depends on (1) collisions that might happen when nodes try to transmit at the same time, (2) the amount of power used to send data, (3) the used protocol.

The main standards protocols in wireless communication are: Wireless Local Area Networks (WLAN/WiFi 802.11), Wireless Personal Area Networks (ZigBee 802.15) and Wireless Metropolitan Area Networks (WiMAX).

The effectiveness of NCSs was proven in many domains such as Swarm Robotics, Multi-Camera Real Time Tracking, Internet and transportation [1] [2] [3] and Mobile Sensor Networks [4].

The contribution of this paper is a study of a H∞ control approach for linear discrete-time Wireless Networked Control Systems with stochastic uncertainty and time-varying packet loss. In this paper, we have study the stability of the closed loop system for WNCS, thus we modelled the packet dropout as, stochastic Bernoulli white sequence parameter and a norm-bounded uncertainties. Then we proceeded to achieve the mean square stability of the uncertain system with random data loss based on an ILMI method.

The paper is organised as follows: Section 1 provides the introduction into the research purpose and presents the basic concepts of WNCS. Section 2 provides the system description and the problem formulation. Section 3 provides the stability analysis of the closed loop system and the controller design. Section 4 illustrates via a numerical example the effectiveness and the validity of the designed controller to improve the H∞ disturbance attenuation performance of the uncertain system in wireless network.

II. System Description and Problem Formulation

A. System Description

Being considered the WNCS with time-varying packet dropouts, the state space representation of the plant is described by:

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$$\begin{cases} x_{k+1} = (A+\Delta A)x_k + Bu_k + B_w w_k = A_\Delta x_k + Bu + B_w w_k \\ ys_k = Cx_k + C_w w_k \\ z_k = Dx_k \end{cases} \quad (1)$$

Where $x_k \in \mathbb{R}^n$ is the state signal, $u_k \in \mathbb{R}^m$ is the control input signal, $w_k \in \mathbb{R}^q$ is the disturbance input signal, $ys_k \in \mathbb{R}^p$ is the measurement output signal and $z \in \mathbb{R}^r$ is the controlled output signal.

The parameter uncertainties ΔA is described as:

$$\|\Delta A\| \le \rho_a \quad (2)$$

The measurement with packet loss and the control input sent over the network are given by:

$$\begin{cases} y_k = (1-\beta) ys_k + \beta y_{k-1} \\ u_k = (1-\alpha) uc_k + \alpha u_{k-1} \end{cases} \quad (3)$$

Where $y_k \in \mathbb{R}^p$ is the output signal, $uc_k \in \mathbb{R}^m$ is the control signal and α and β are a mutually independent stochastic variables that follow Bernoulli distributed white sequence [5]:

$$Prob(x=i) = \begin{cases} \mathbb{E}(x) = \bar{x} & for \quad i=0 \\ 1-\mathbb{E}(x) = 1-\bar{x} & for \quad i=1 \end{cases} \quad (4)$$

The WNCS representation for uncertain system with data loss is derived from *(1) to (4)* and given by:

$$\begin{cases} x_{k+1} = (A+\Delta A)x_k + Bu_k + B_w w_k \\ ys_k = Cx_k + C_w w_k \\ u_k = (1-\alpha)uc_k + \alpha u_{k-1} \\ y_k = (1-\beta) ys_k + \beta y_{k-1} \end{cases} \quad (5)$$

B. Problem formulation

In order to study the H∞ stability of the above uncertain system, we have defined for the augmented form of the system (6) and the dynamic controller (7) such as:

$$\begin{cases} \tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}uc_k + \tilde{B}_w w_k \\ y_k = \tilde{C}\tilde{x}_k + \tilde{C}_w w_k \\ z_k = \tilde{D}\tilde{x}_{k+1} \end{cases} \quad (6)$$

$$\begin{cases} \hat{x}_{k+1} = A_{ctr}\hat{x}_k + B_{ctr} y_k \\ uc_k = C_{ctr}\hat{x}_k \end{cases} \quad (7)$$

Where:

$$\tilde{x}_{k+1} = \begin{bmatrix} x_k \\ u_{k-1} \\ y_{k-1} \end{bmatrix}; \tilde{A} = \begin{bmatrix} A+\Delta A & \alpha B & 0 \\ 0 & \alpha I & 0 \\ (1-\beta)C & 0 & \beta I \end{bmatrix}; \tilde{B} = \begin{bmatrix} (1-\alpha)B \\ (1-\alpha)I \\ 0 \end{bmatrix};$$

$$\tilde{B}_w = \begin{bmatrix} B_w \\ 0 \\ (1-\beta)C_w \end{bmatrix}; \tilde{C} = \begin{bmatrix} (1-\beta)C & 0 & \beta I \end{bmatrix};$$

$$\tilde{C}_w = (1-\beta)C_w; \tilde{D} = \begin{bmatrix} D & 0 & 0 \end{bmatrix}$$

A_{ctr}, B_{ctr} and C_{ctr} C_{ctr} are the controller matrices to be determined. For that purpose, we define the discrete time-varying system *(8)* with stochastic parameters α and β:

$$\begin{cases} X_{k+1} = \begin{bmatrix} \tilde{x}_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}C_{ctr} \\ B_{ctr}\tilde{C} & A_{ctr} \end{bmatrix} X_k + \begin{bmatrix} \tilde{B}_w \\ B_{ctr}\tilde{C}_w \end{bmatrix} w_k \\ z_k = \begin{bmatrix} \tilde{D} & 0 \end{bmatrix} \ddot{X}_k \end{cases} \quad (8)$$

We proceed with the segregation of stochastic parameters in (8) into stochastic part and deterministic part, we obtain:

$$\begin{cases} X_{k+1} = (\bar{A} + \bar{\Delta}A) X_k + \left((\alpha - \bar{\alpha}) \bar{A}_\alpha + (\beta - \bar{\beta}) \bar{A}_\beta \right) X_k \\ \qquad + \bar{B}_W w_k + (\beta - \bar{\beta}) \bar{B}_\beta w_k \\ z_k = \bar{D} X_k \end{cases} \quad (9)$$

Where:

$$\bar{A} = \begin{bmatrix} A_1 & B_1 C_{ctr} \\ B_{ctr} C_1 & A_{ctr} \end{bmatrix}, \bar{A}_\alpha = \begin{bmatrix} A_\alpha & -B_\alpha C_{ctr} \\ 0 & 0 \end{bmatrix}$$

$$\bar{A}_\beta = \begin{bmatrix} A_\beta & 0 \\ B_{ctr} C_\beta & 0 \end{bmatrix}, \bar{B}_W = \begin{bmatrix} B_w \\ B_{ctr} C_w \end{bmatrix},$$

$$\bar{B}_\beta = \begin{bmatrix} B_{\beta w} \\ -B_{ctr} C_w \end{bmatrix}, \bar{D} = \begin{bmatrix} D & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} A & \bar{\alpha} B & 0 \\ 0 & \bar{\alpha} I & 0 \\ (1-\bar{\beta})C & 0 & \bar{\beta} I \end{bmatrix}; B_1 = \begin{bmatrix} (1-\bar{\alpha})B \\ (1-\bar{\alpha})I \\ 0 \end{bmatrix};$$

$$A_\alpha = \begin{bmatrix} 0 & B & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}; B_\alpha = \begin{bmatrix} B \\ I \\ 0 \end{bmatrix}; A_\beta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C & 0 & I \end{bmatrix};$$

$$B_\beta = \begin{bmatrix} 0 \\ 0 \\ -C_w \end{bmatrix}; C_1 = \begin{bmatrix} (1-\bar{\beta})C & 0 & \bar{\beta} I \end{bmatrix}; C_\beta = \begin{bmatrix} -C & 0 & I \end{bmatrix}$$

III. Stability Analysis and Control Design

In order to study the stability of the uncertain stochastic system *(9)* we proceed with the design of the controller (7) such that the closed loop system satisfies the two requirements mentioned here below:

1- The H∞ mean square stochastic stability of the closed loop system.
2- The H∞ performance disturbance attenuation under zero initial condition

A. The H∞ mean square stochastic stability

The following theorem gives sufficient condition for the H∞ mean square stochastic stability for the studied WNCS.

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Theorem 1: The system *(9)* is mean square stable if there exist a positive matrix P satisfying the LMI (10).

Proof: by defining the Lyapunov function $V_k = X_k^T P X_k$ where P is a unique positive definite matrix, it follows from *(9)* that:

$$\begin{aligned}&\mathbb{E}\left\{V\left(X_{k+1}\right) \mid X_k,..,X_0\right\}-V\left(X_k\right)\\&=\mathbb{E}\left\{X_{k+1}^T P X_{k+1}\right\}-X_k^T P X_k = X_k{}^T \Phi\, X_k\end{aligned} \tag{11}$$

Where:

$$\begin{aligned}\Phi &= \left(\bar{A}+\bar{\Delta}A\right)^T P\left(\bar{A}+\bar{\Delta}A\right)+\left(\bar{\alpha}-\bar{\alpha}^2\right)\bar{A}_\alpha{}^T P\bar{A}_\alpha\\&+\left(\bar{\beta}-\bar{\beta}^2\right)\bar{A}_\beta{}^T P\bar{A}_\beta - P\end{aligned} \tag{12}$$

By applying Schur complement, we conclude that $\Phi < 0$ if and only if (10) holds.

$\Phi < 0$ Then:

$$X_k{}^T \Phi\, X_k \le -\theta(-\Phi) X_k{}^T X_k < -\delta X_k{}^T X_k < -\frac{\delta}{\gamma} V\left(X_k\right) \tag{13}$$

Where $0 < \delta < \min(\theta_{\min}(-\Phi), \theta_{\max}(P))$

From (13), we infer that:

$$\mathbb{E}\left\{V\left(X_{k+1}\right) \mid X_k, X_{k-1},\ldots, X_0\right\} \le \left(1-\frac{\delta}{\gamma}\right)^k V\left(X_0\right) \tag{14}$$

From (14) and using results in [6] we come to the conclusion that the system (9) is mean square stable. Then the proof is accomplished .

B. *Robust controller design*

In this section we investigate the H∞ performance disturbance attenuation for the closed loop system (9). The main result is formulated in the following theorem.

Theorem 2: The controller (7) is designed so that the closed loop system (9) is exponentially mean square stable and the robust disturbance attenuation, under zero initial condition, is achieved if there exists a positive definite matrix P and a scalar $\varphi > 0$ satisfying LMI (15).

Where $\surd_\alpha = \sqrt{\left(\bar{\alpha}-\bar{\alpha}^2\right)}$ and $\surd_\beta = \sqrt{\left(\bar{\beta}-\bar{\beta}^2\right)}$

Proof: if for any non-zero w_k, we define:

$$\mathbb{E}\left\{V\left(X_{k+1}\right)\right\}-\mathbb{E}\left\{V\left(X_k\right)\right\}+\mathbb{E}\left\{z_k^T z_k\right\}-\varphi^2\mathbb{E}\left\{w_k^T w_k\right\} \tag{16}$$

which can rewritten as:

$$\mathbb{E}\left\{\begin{bmatrix}X_k\\ w_k\end{bmatrix}^T \Lambda \begin{bmatrix}X_k\\ w_k\end{bmatrix}\right\} \tag{17}$$

where:

$$\Lambda = \begin{bmatrix}\Lambda_1 & \Lambda_2{}^T\\ \Lambda_2 & \Lambda_3\end{bmatrix}$$

$$\begin{aligned}\Lambda_1 &= \left(\bar{A}+\bar{\Delta}_A\right)^T P\left(\bar{A}+\bar{\Delta}_A\right)+\left(\bar{\alpha}-\bar{\alpha}^2\right)\bar{A}_\alpha{}^T P\bar{A}_\alpha\\&+\left(\bar{\beta}-\bar{\beta}^2\right)\bar{A}_\beta{}^T P\bar{A}_\beta+\bar{D}^T\ \bar{D}-P\end{aligned}$$

$$\Lambda_2 = \bar{B}_w{}^T P\left(\bar{A}+\bar{A}_A\right)+\left(\bar{\beta}-\bar{\beta}^2\right)\bar{B}_\beta{}^T P\bar{A}_\beta$$

$$\Lambda_3 = \bar{B}_w{}^T P\bar{B}_w+\left(\bar{\beta}-\bar{\beta}^2\right)\bar{B}_\beta{}^T P\bar{B}_\beta-\varphi^2 I$$

The function in (16) is definite negative if:

$$\Lambda = \begin{bmatrix}\Lambda_1 & \Lambda_2{}^T\\ \Lambda_2 & \Lambda_3\end{bmatrix} < 0 \tag{18}$$

By applying Schur Complement on (18), we get the LMI (15). Hence, from (16), (17) and (18) we conclude that:

$$\mathbb{E}\left\{V\left(X_{k+1}\right)\right\}-\mathbb{E}\left\{V\left(X_k\right)\right\}+\mathbb{E}\left\{z_k^T z_k\right\}-\varphi^2\mathbb{E}\left\{w_k^T w_k\right\} < 0 \tag{19}$$

By summing up (19) from 0 to ∞ we find:

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|z_k\right\|^2\right\} < \varphi^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|w_k\right\|^2\right\}+\mathbb{E}\left\{V_0\right\}-\mathbb{E}\left\{V_\infty\right\} \tag{20}$$

Which achieves the proof.

$$\begin{bmatrix}-P & \bar{A}^T+\bar{\Delta}_A{}^T & \bar{A}_\alpha{}^T & \bar{A}_\beta{}^T\\ \bar{A}+\bar{\Delta}_A & -P^{-1} & 0 & 0\\ \bar{A}_\alpha & 0 & -\left(\bar{\alpha}-\bar{\alpha}^2\right)^{-1}P^{-1} & 0\\ \bar{A}_\beta & 0 & 0 & -\left(\beta-\bar{\beta}^2\right)^{-1}P^{-1}\end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix}-P & 0 & \left(\bar{A}+\bar{\Delta}_A\right)^T P & \surd_\alpha \bar{A}_\alpha{}^T P & \surd_\beta \bar{A}_\beta{}^T P & \bar{D}^T\\ 0 & -\varphi^2 I & \bar{B}_w{}^T P & 0 & \surd_\beta \bar{B}_\beta{}^T P & 0\\ P\left(\bar{A}+\bar{\Delta}_A\right) & Q\bar{B}_w & -P & 0 & 0 & 0\\ \surd_\alpha P\bar{A}_\alpha & 0 & 0 & -P & 0 & 0\\ \surd_\beta P\bar{A}_\beta & \surd_\beta P\bar{B}_\beta & 0 & 0 & -P & 0\\ \bar{D} & 0 & 0 & 0 & 0 & -I\end{bmatrix} < 0 \tag{15}$$

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C. The uncertainty conversation

Based on the obtained result of theorem 1 and theorem 2, and the S-procedure theorem in [7] we can easily deduce the LMI *(21)*.

In order to derive the controller parameters, we apply a suitable congruence transformation of (21). For this goal, the matrix P is partitioned as:

$$P=\begin{bmatrix} E & G \\ G^T & \varnothing \end{bmatrix}=\begin{bmatrix} F & J \\ J^T & \varnothing \end{bmatrix}^{-1}>0 \tag{22}$$

Where E , F are $\ell\times\ell$ symmetric positive-definite matrices; G, J are non-singular matrices and $\varnothing$ denotes matrix block that is irrelevant for the derivation of controller parameters.

From $PP^{-1}=I$ we conclude that the transformation matrices $\Psi_F=\begin{bmatrix} F & I \\ J^T & 0 \end{bmatrix}$, $\Psi_E=\begin{bmatrix} I & E \\ 0 & G^T \end{bmatrix}$ satisfying :

$$P\Psi_{\mathrm{F}}=\Psi_{\mathrm{E}};\ \Psi_{\mathrm{F}}{}^{\mathrm{T}}P=\Psi_{\mathrm{E}}{}^{\mathrm{T}} \tag{23}$$

We performed a first congruence transformation of (21) with $\Gamma=diag\left(\Psi_F,I,\Psi_F,\Psi_F,\Psi_F,I,I,I\right)$ on the right and Γ^T on the left with considering equalities (23). A second congruence transformation with $\Gamma=diag\left(F^{-1},I,I,I,E^{-1},I,E^{-1},I,E^{-1},I,I,I\right)$ is then applied on the right and on the left, to give the LMI (24).

Where :

$$\begin{cases}\Gamma_1=C_{ctr}J^TF^{-1},\ \Gamma_2=ZGA_{ctr}J^TF^{-1}\\ \Gamma_3=ZGB_{ctr},\qquad Z=E^{-1}\end{cases} \tag{25}$$

The feasibility of (24) is conditioned by the non-singular matrices J , G chosen such that $GJ^T=I-EF$

Consequently, the controller parameters in (7) can be deduced as follows:

$$\begin{cases}A_{ctr}=G^{-1}Z^{-1}\Gamma_2\left(ZF^{-1}-I\right)^{-1}ZG\\ B_{ctr}=G^{-1}Z^{-1}\Gamma_3\\ C_{ctr}=\Gamma_1\left(ZF^{-1}-I\right)ZG\end{cases} \tag{26}$$

By applying a linear transformation $\tilde{x}_k=ZG\hat{x}_k$, we get a new representation of the controller (7):

$$\begin{cases}\tilde{x}_{k+1}=\tilde{A}_{ctr}\tilde{x}_k+\tilde{B}_{ctr}y_k\\ u_k^{ctr}=\tilde{C}_{ctr}\tilde{x}_k\end{cases} \tag{27}$$

where :

$$\begin{cases}\tilde{A}_{ctr}=\Gamma_2\left(ZF^{-1}-I\right)^{-1}\\ \tilde{B}_{ctr}=\Gamma_3\\ \tilde{C}_{ctr}=\Gamma_1\left(ZF^{-1}-I\right)\end{cases}$$

$$\begin{bmatrix}
-P & 0 & \bar{A}^TP & \sqrt{\alpha}\,\bar{A}_\alpha{}^TP & \sqrt{\beta}\,\bar{A}_\beta{}^TP & \bar{D}^T & 0 & \lambda I\\
0 & -\varphi^2I & \bar{B}_w{}^TP & 0 & \sqrt{\beta}\,\bar{B}_\beta{}^TP & 0 & 0 & 0\\
P\bar{A} & P\bar{B}_w & -P & 0 & 0 & 0 & \rho_aP & 0\\
\sqrt{\alpha}\,P\bar{A}_\alpha & 0 & 0 & -P & 0 & 0 & 0 & 0\\
\sqrt{\beta}\,P\bar{A}_\beta & \sqrt{\beta}\,P\bar{B}_\beta & 0 & 0 & -P & 0 & 0 & 0\\
\bar{D} & 0 & 0 & 0 & 0 & -I & 0 & 0\\
0 & 0 & \rho_aP & 0 & 0 & 0 & -\lambda I & 0\\
\lambda I & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda I
\end{bmatrix}<0 \tag{21}$$

$$\begin{bmatrix}
-F^{-1} & * & * & * & * & * & * & * & * & * & * & *\\
-F^{-1} & -E & * & * & * & * & * & * & * & * & * & *\\
0 & 0 & -\varphi^2I & * & * & * & * & * & * & * & * & *\\
A_1+B_1\Gamma_1 & A_1 & B_2 & -F & * & * & * & * & * & * & * & *\\
A_1+\Gamma_3C_1+B_1\Gamma_1+\Gamma_2 & A_1+\Gamma_3C_1 & B_2+\Gamma_3C_2 & -Z & -Z & * & * & * & * & * & * & *\\
\sqrt{\alpha}\left(A_\alpha-B_\alpha\Gamma_1\right) & \sqrt{\alpha}\,A_\alpha & 0 & 0 & 0 & -F & * & * & * & * & * & *\\
\sqrt{\alpha}\left(A_\alpha-B_\alpha\Gamma_1\right) & \sqrt{\alpha}\,A_\alpha & 0 & 0 & 0 & -Z & -Z & * & * & * & * & *\\
\sqrt{\beta}\,A_\beta & \sqrt{\beta}\,A_\beta & \sqrt{\beta}\,B_\beta & 0 & 0 & 0 & 0 & -F & * & * & * & *\\
\sqrt{\beta}\left(A_\beta+\Gamma_3C_\beta\right) & \sqrt{\beta}\left(A_\beta+\Gamma_3C_\beta\right) & \sqrt{\beta}\left(B_\beta-\Gamma_3C_W\right) & 0 & 0 & 0 & 0 & -Z & -Z & * & * & *\\
D_d & D_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & *\\
0 & 0 & 0 & \rho_aF & \rho_aZ & 0 & 0 & 0 & 0 & 0 & -\lambda I & *\\
\lambda I & \lambda I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda I
\end{bmatrix}<0 \tag{24}$$

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We proceed to turn all constraints into LMI (24), for that we note:

$$S = F^{-1} \quad (28)$$

To solve the obtained LMI, we have to solve the problem below:

$$\min \varphi \quad (29)$$
$$\text{Subject to } (24), (25) \text{ and } (28)$$

For the variable φ, *and the symmetric positive definite matrices* $E, F, S, Z, \Gamma_1, \Gamma_2, \Gamma_3$ *and the positive scalar* γ

To solve the abovementioned non-convex optimization problem, we have adopted the SLPMM (Sequential Linear Programming Matrix Method) [8] and the SDP (Semi-Definite Programming) for the relaxation of the constraints:

Then $S = F^{-1}$ and $Z = E^{-1}$ holds if and only if : $\begin{bmatrix} S & I \\ I & F \end{bmatrix} \succ 0$, $\begin{bmatrix} Z & I \\ I & E \end{bmatrix} \succ 0$, $Trace(SF) \geq \ell$ and $Trace(ZE) \geq \ell$ hold.

The non-convex problem (29) can be resolved by finding a feasible solution for the above problem:

$$\min\left(Trace(SF) + Trace(ZE) + \varphi^2\right) \quad (30)$$
$$\text{Subject to } (24)\ (25) \text{ and } (28)$$

For the variable φ, *and the symmetric positive definite matrices* $E, F, S, Z, \Gamma_1, \Gamma_2, \Gamma_3$ *and the positive scalar* γ

We propose the following ILMI (iterative LMI) algorithm to solve (30) in order to achieve the computation of the parameters of the studied controller:

1- Find the initial point E^0, F^0, S^0, Z^0 such that the LMIs (24) hold and set $\varphi_{\min} = \varphi$

2- Find $E^k, F^k, S^k, Z^k, \Gamma_1, \Gamma_2, \Gamma_3, \lambda$ by solving:

$$\min\left(Trace\left(SF^k + S^kF\right) + Trace\left(ZE^k + Z^kE\right) + \varphi^2\right)$$
$$\text{Subject to } (24)\ (25) \text{ and } (28)$$

3 - If $\left|Trace\left(SF^k + S^kF\right) + Trace\left(ZE^k + Z^kE\right) + \varphi^2\right| \leq 4\ell$

then $\varphi_{\min} = \min\{\varphi, \varphi_{\min}\}$. The solution is $E, F, S, Z, \Gamma_1, \Gamma_2, \Gamma_3, \lambda, \varphi_{\min}$ and compute the controller parameters (26)

Else k= k+1 then go to step 2, until finding a feasible solution.

IV. Simulation Result

In this section we propose the application of the prosed control approach to a numerical example in order to validate the presented theoretical results. System parameters are given by:

$$A = \begin{bmatrix} 0.9226 & -0.6330 & 0 \\ 1 & -0.44 & 0 \\ 0 & 1 & -0.42 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \\ 0 & 0.61 \end{bmatrix}, C = \begin{bmatrix} 4.738 & 5.287 & 0 \\ 2.14 & 0 & 12.287 \end{bmatrix}$$

$$B_W = \begin{bmatrix} 0.5 \\ 0.1 \\ 0.2 \end{bmatrix}, C_W = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \rho_a = 0.01$$

The simulation results are carried out using MATLAB YALMIP [9] , the SeDuMi solver [10] and the Truetime simulator [11].

Based on random communication delay $\bar{\alpha} = \bar{\beta} = 0.1$, with 10^{-3} accuracy, the $H\infty$ performance is $\varphi_{\min} = 0.448$, this result is obtained in 87 iterations.

The closed loop state trajectories under wireless network ZigBee (IEEE 802.15.4) are illustrated in the Fig. 1.

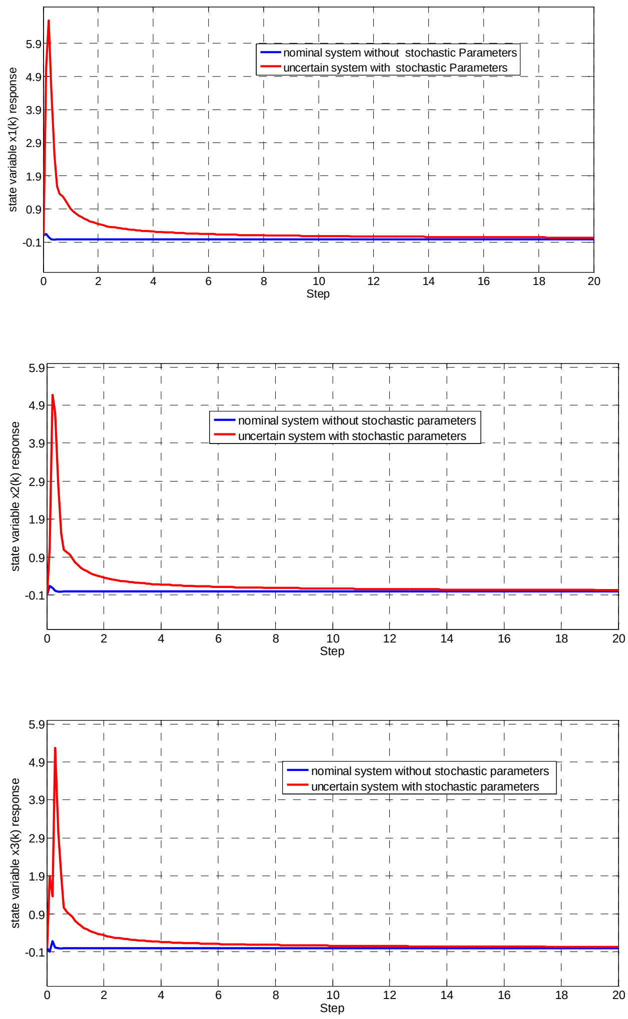

Fig. 1. The closed loop state trajectories of the system controlled via the wireless network

It is obvious from the state space trajectories of the closed loop system that the system is exponentially mean square stable, which prove the validity of the adopted approach.

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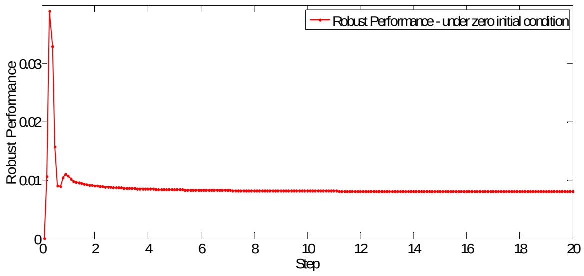

Fig. 3. Robust performance under zero initial condition

The simulation result under null initial conditions is presented in Fig.2. By noting that $\varphi_{\min} = 0.448$, the obtained result shows that the H∞ performance is achieved.

V. CONCLUSIONS

This paper proposes the design of a robust $H\infty$ dynamic feedback controller in order to stabilize Wireless Networked Controlled Systems which is subject to uncertainty and stochastic data loss. The stability is studied in the sense of mean square stability. Simulation results via numerical example shows the behaviour of the WNCS and have demonstrated that the proposed approach is relevant.

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