Discrete Neuro Sliding Mode Control for Uncertain Nonlinear Systems

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Abstract— This paper proposes a discrete-time neuro-sliding mode control for an uncertain nonlinear system (NL_NDSMC). The proposed technique guarantees the stability of the system and achieves zero tracking error comparing with the classical discrete sliding mode control. Although, it reduces chattering phenomenon. The selection of the neuronal sliding surface is an important issue, which have been studied carefully. Furthermore, simulations are carried out on an inverted pendulum with and without uncertainties. The obtained results confirm the efficiency of the proposed approach.

Keywords— Discrete-time, sliding mode control, Neuronal Approach, Nonlinear System, Uncertainties, Chattering phenomenon

I. INTRODUCTION

Technological development increases the complexity of industrial processes, which can be presented in different aspects, such as strong non-linearity, non-stationarity and the wide range of operations. The need of new control characterized by its robustness with respect to the system's modelling becomes a goal to attempts by most researches.

Many control techniques have been widely studied on nonlinear system as those dealing with artificial intelligence such as fuzzy [1], [2] neuronal [3] and genetic algorithm [4], [5].

The variable structure control (VSC) [6], is also one of the most attractive control research areas dealing with linear/ nonlinear, continuous/discrete with or without uncertainties, time delay systems [7], [8]. A particular case of VSC is the sliding mode control (SMC). This technique is known as one of the robust, insensitive to parameter variations and fast dynamic response control technique [9], [10], [11], [12].

The principal inconvenient of the SMC is the chattering phenomenon which appears as a source to excite unmodeled high frequency dynamics of the process. This chattering, caused by the discontinuous part of the sliding control, consists of oscillations around the sliding surface leading to adverse effects on the system. The knowledge of the dynamics of the system is another inconvenient in the calculation of the equivalent part of the control [10].

In the literature, some suggestions have been presented to overcome these two main problems in both linear and nonlinear systems dealing with continuous and discrete time [13], [14].

The most popular technique for eliminating chattering was the substitution of the sign function by the saturation in the discontinuous control [10], [11].

In recent years, in order to improve the SMC performance and to offset its disadvantages, various sliding mode controllers are proposed to control continuous nonlinear system. We can cite fuzzy sliding mode control [15], neural network sliding mode control [16] and genetic sliding mode control [17]. With the improve of calculator researches, a discrete form of fuzzy sliding mode control [18] and genetic SMC are developed. However, the discrete neuro sliding mode control for nonlinear system remains an important research area.

In this paper, a new discrete-time neuro-sliding mode control for uncertain nonlinear systems (NL_NDSMC) is proposed. Comparing to the result of the classical discrete sliding mode control (DSMC), this one shows a good tracking error and reducing the chattering phenomenon.

The paper is organized as follows. An overview of the discrete SMC technique for nonlinear system is presented in the second section. A neuronal sliding mode controller design is proposed in third section. The final section is dedicated to the simulation results of the DSMC on the inverted pendulum in two cases (certain and uncertain) as well as the simulation results of the proposed NL-NDSMC for the purpose of performance comparison. A conclusion summarizes the present paper.

II. DISCRETE SLIDING MODE CONTROL: DSMC

We consider the nonlinear discrete time system described by:

$$x(k+1) = f(x(k)) + g(x(k))u(k)$$
(1)

where:

 $x(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T \in \Re^n$ is the state vector,

f and g are two nonlinear functions and u(k) is the input signal.

The first step to design a sliding mode control is to determine the sliding hyperplane with desired dynamics of the corresponding sliding motion. The next step is to design the control input so that the state trajectories are driven and attracted toward the sliding hyperplane and then remained sliding on it for all subsequent time [18].

Otherwise, the discrete sliding function S(k) is chosen as [9], [19]:

$$S(k) = C.x(k) \tag{2}$$

with $C \in \mathbb{R}^{1 \times n}$.

The Reaching law method is choosing as [20]:

$$S(k+1) = \varphi S(k) - M' sign(S(k))$$
(3)

where *sign* is the signum function defined as:

$$sign(S(k)) = \begin{cases} -1 & if \qquad S(k) < 0\\ 1 & if \qquad S(k) > 0 \end{cases}, \varphi \in \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Using (1), (2) and (3), we obtain:

$$S(k+1) = C.x(k+1)$$

= $C[f(x(k)) + g(x(k))u(k)]$
= $C.f(x(k)) + C.g(x(k))u(k)$
= $\varphi S(k) - M'sign(S(k))$ (4)

We suppose that g(x(k)) is inversible, then the discrete sliding mode control for nonlinear system can be expressed as:

$$u(k) = \left(C.g\left(x(k)\right)\right)^{-1} \left[-C.f\left(x(k)\right) + \varphi S(k) - M'sign\left(S(k)\right)\right]$$
(5)

III. THE NEURONAL DISCRETE SLIDING MODE CONTROLLER

We consider that the nonlinear system is submitted to uncertainties. Then, the model of nonlinear uncertain system is defined as:

$$x(k+1) = F_u(x(k)) + G_u(x(k))u(k)$$
(6)

where:

$$F_{u}(x(k)) = f(x(k) + \Delta f(x(k)))$$

$$G_{u}(x(k)) = g(x(k) + \Delta g(x(k)))$$
(7)

 $\Delta f(x(k))$ and $\Delta g(x(k))$ are the uncertainties on f(x(k)) and g(x(k)) respectively.

In order to increase the performances and eliminate chattering of DSMC, combining sliding mode control with other robust technique such as neural networks (NN) control appeared to be an interesting concept. Many researchers have published various control scheme based on this idea [21], [22], [23] and [24].

Some works attempts to apply a neuronal network NN as an observer in the estimation of equivalent control [21]. Also, in [22], a sliding mode controller with a modified switching function that produces a low-chattering control is used in parallel with an artificial NN for online identification of the model errors, which imposes the controller performances [23]. Here, we propose, a neuron online estimation of the errors in the sliding function in order to improve the DSMC performance.

The neural network NN used in this estimation is a reduced form of the multi-layer perception (MLP, multilayer perception). This architecture of the network is most commonly used with the back-propagation algorithm as seen in Figure 1.



Fig. 1 The feed-forward neural network of the NL-NDSMC

To estimate the parameters of the network, we use the algorithm of back propagation. This algorithm is generally more efficient than others in terms of number of arithmetic operations to be performed to evaluate the gradient of a cost function. This function can be expressed as:

$$J(\underline{W}, \underline{B}, \underline{C}) = \sum_{k=1}^{N} \left[x(k) - x_{ref}(k) \right]^{2}$$
(8)

where x_{ref} is the reference state vector.

we note $e(k) = x(k) - x_{ref}(k)$

The output of the neural network at a given time is in the form:

$$x_{n}(k) = \sum_{j=1}^{n} \sum_{k=1}^{q} c_{k} f(w_{kj} x_{j}(k) + b_{kj})$$
(9)

where:

 w_{kj} and b_k (for k = 1...q and j = 1...n; q is the number of neuron in the hidden layer and n is the number of inputs)are respectively the weights and biases of the layer of input neurons, c_k are the weights of output of the hidden layer.

The network learning is based on minimizing the quadratic criterion $J(\underline{W}, \underline{B}, \underline{C})$ which relates the error e(k) whose analytical expression is of the form:

$$e(k) = \sum_{j=1}^{n} \sum_{k=1}^{m} c_k f(w_{kj} x_j + b_k) - x_{ref}(k)$$
(10)

The readjustment of the weight of the synoptic network is then accomplished by the gradient method [25], we have:

$$w_{kj}(k+1) = w_{kj}(k) - \mu \frac{\partial e^2(k)}{\partial w_{kj}j(k)} = w_{kj}j(k) - 2\mu e(k)\frac{\partial e(k)}{\partial w_{kj}j(k)}$$

= $w_{kj}(k) + 2\mu e(k)c_k(k)x_j(k)f'(w_{kj}j(k)x_j(k) + b_k(k))$
(11)

$$b_{k}(k+1) = b_{k}(k) - \mu \frac{\partial e^{2}(k)}{\partial b_{k}(k)} = b_{k}(k) + 2\mu e(k)f'(w_{kj}(k)x_{j}(k) + b_{k}(k))$$

(12)

$$c_{k}(k+1) = c_{k}(k) - \mu \frac{\partial e^{2}(k)}{\partial c_{k}(k)} = c_{k}(k) + 2\mu e(k)f(w_{kj}(k)x_{j}(k) + b_{k}(k))$$
(13)

where μ is the coefficient of learning, which must be chosen to ensure the escalation between learning and the speed of parametric convergence.

The neural network tries to approximate a nonlinear relationship between the real process and the local model which is based on the errors taking into account that the neural networks are universal approximators. The learning algorithm is based on the optimization method of Levenberg-Marquardt.

The main incentive choice of the Levenberg-Marquardt algorithm rests on the fast guarantee of the convergence toward a minimum. The step developed here makes it possible to integrate the whole of knowledge available on the error provided by the neural network [25], [26].

Then we obtain the following neuronal discrete sliding mode controller for nonlinear system as:

$$u_n(k) = \left(C.g\left(x_n(k)\right)\right)^{-1} \left[-C.f\left(x_n(k)\right) + \varphi S_n(k) - M'sign\left(S_n(k)\right)\right]$$
(14)
where: x_n is the output of the neural network and S_n the new
sliding surface defined as:

$$S_n(k) = C.x_n(k) \tag{15}$$

IV. SIMULATION RESULTS

To evaluate the robustness of the proposed neuronal discrete sliding mode control, we consider a nonlinear system which consist on an inverted pendulum model.

A. System description

The inverted pendulum is often used as a benchmark for all kinds of controllers. It is a nonlinear, unstable system which makes it challenge to control. The system is composed of a rigid pole and a cart on which the pole is hinged to the cart through a pivot such that it has only one degree of freedom. The goal of the control is to make the pole upright. The dynamic model equations for the invented pendulum are [18]:

$$M\ddot{x} + N = u \tag{16}$$

$$N = m\ddot{x} + ml\theta\cos\theta - ml\theta^2\sin\theta \tag{17}$$

$$P - mg = -ml(\dot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \tag{18}$$

$$I\ddot{\theta} = Pl\sin\theta - Nl\cos\theta \tag{19}$$

where *M* is the mass of the cart, m is the mass of the pendulum, $I = (1/3) ml^2$ is the moment of the inertia of the pendulum, θ is the angular position of the pendulum deviated from the equilibrium position, *x* is the position of the cart, *l* is the half length of the pendulum. The system friction is omitted for simplicity [18].

The dynamic equation of θ can be rewritten as:

$$\left[(M+m)(ml^2+I) - (ml\cos\theta)^2 \right] \ddot{\theta} + (ml\dot{\theta})^2\cos\theta\sin\theta -$$
(20)
$$(M+m)ml\sin\theta + ml\cos\theta = 0$$

If we define $x_1 = \theta$ and $x_2 = \dot{\theta}$, the state representation of the considered system can be written as:

$$\begin{cases} \dot{x}_{1}(t) = x_{2}(t) \\ \dot{x}_{2}(t) = f_{c}(x_{1}(t), x_{2}(t)) + g_{c}(x_{1}(t), x_{2}(t)) u(t) \\ y(t) = x_{1}(t) \end{cases}$$
(21)

with f_c and g_c are two nonlinear continuous functions.

The discrete model of the inverted pendulum can be expressed as:

$$x(k+1) = \begin{bmatrix} x_{1}(k+1) \\ x_{2}(k+1) \end{bmatrix} = \begin{bmatrix} x_{1}(k) \\ x_{2}(k) \end{bmatrix} + T \begin{bmatrix} x_{2}(k) \\ f_{c}(x_{1}(k), x_{2}(k)) \end{bmatrix} + T \begin{bmatrix} 0 \\ g_{c}(x_{1}(k), x_{2}(k)) \end{bmatrix} u(k)$$

= $f(x(k)) + g(x(k))u(k)$
(22)

22) 1.1.7

with *T* is the sampling rate, and:

$$f(x(k)) = \begin{bmatrix} f_1(x(k)) \\ f_2(x(k)) \end{bmatrix} = \begin{bmatrix} x_1(k) + T \cdot x_2(k) \\ x_2(k) + T \cdot f_c(x_1(k), x_2(k)) \end{bmatrix}$$
(23)
with $f_c(x_1(k), x_2(k)) = F_1(k) - F_2(k)$, and:

$$F_{1}(k) = \frac{(M+m).(m.l.g.\sin(x_{1}(k)))}{(M+m).(m.l^{2}+I) - (m.l.\cos(x_{1}(k)))^{2}}$$
(24)

$$F_{2}(k) = \frac{(ml.x_{1}(k))^{2}.\cos(x_{1}(k)).\sin(x_{1}(k))}{(M+m).(ml^{2}+I) - (ml.\cos(x_{1}(k)))^{2}}$$
(25)

as well as:

$$g(x(k)) = \begin{bmatrix} g_1(x(k)) \\ g_2(x(k)) \end{bmatrix} = \begin{bmatrix} 0 \\ Tg_c(x_1(k), x_2(k)) \end{bmatrix}$$
(26)

with:

$$g_{c}(x_{1}(k), x_{2}(k)) = \frac{ml.\cos(x_{1}(k))}{(M+m).(ml^{2}+l) - (ml.\cos(x_{1}(k)))^{2}}$$
(27)

Then, we obtain:

$$\begin{cases}
x_1(k+1) = x_1(k) + Tx_2(k) \\
x_2(k+1) = x_2(k) + T(F_1(k) - F_2(k) - Tg_c(k)u(k)) \\
y(k) = x_1(k)
\end{cases}$$
(28)

The parameters of the inverted pendulum are given as:

l = 0.5m: the half-length of the pendulum,

 $I = (1/3) ml^2$: the moment of the inertia of the pendulum,

m = 0.3Kg: the mass of the pendulum,

M = 2kg: the mass of the cart,

 $g = 9.81 m/s^2$: the gravity.

The sampling rate is chosen as T = 0.1s.

B. The DSMC for the inverted pendulum:

Firstly, we apply a discrete SMC for the inverted pendulum. The synthesis parameters are chosen as:

$$C = \begin{bmatrix} 0.5\\1 \end{bmatrix}, \qquad x_{ref} = \begin{bmatrix} 0\\0 \end{bmatrix}$$

The simulation results with the NL-DSMC for the inverted pendulum is shown in Figures 2, 3 and 4. Figure 2 shows the evolution of the state $x_1(k)$ and $x_2(k)$ Figure 3 shows the evolution of the controller and Figure 4 presents the evolution of the sliding function.







It obvious that the classical state vector using the discrete sliding mode control (DSMC) converges to zero but cannot remove the chattering phenomenon.

C. The DSMC for the uncertain inverted pendulum:

In this study, the nonlinear uncertain system is defined as (6) and (7). Then the state model of the discrete system can be written as:

$$\begin{cases} x_1(k+1) = x_1(k) + Tx_2(k) \\ x_2(k+1) = x_2(k) + T((F_{1u}(k) - F_{2u}(k)) - G_{2u}(k)u(k)) \\ y(k) = x_1(k) \end{cases}$$
(29)

where:

$$F_{1u}(k) = \frac{((M + \Delta M) + (m + \Delta m)).((m + \Delta m)J.g.\sin(x_1(k)))}{((M + \Delta M)) + (m + \Delta m)).((m + \Delta m)J^2 + I) - ((m + \Delta m)J.\cos(x_1(k)))^2}$$

$$F_{2u}(k) = \frac{((m + \Delta m)J.x_1(k))^2.\cos(x_1(k)).\sin(x_1(k))}{((M + \Delta M)) + (m + \Delta m)).((m + \Delta m)J^2 + I) - ((m + \Delta m)J.\cos(x_1(k)))^2}$$
(30)

as well as:

$$G_{2u}(k) = \frac{(m + \Delta m)J.\cos(x_1(k))}{((M + \Delta M) + (m + \Delta m)).((m + \Delta m)J^2 + I) - ((m + \Delta m)J.\cos(x_1(k)))^2}$$
(31)

The simulation results of the discrete nonlinear sliding mode control are shown in figures 5, 6 and 7. Figure 5 present the evolution of the states $x_1(k)$ and $x_2(k)$. Figure 6 illustrate the evolution of the controller u(k) and figure 7 shows the evolution of the sliding function S(k).



Fig. 5 The Evolution of states $x_1(k)$ and $x_2(k)$ (uncertain nonlinear system)



Fig. 6 Evolution of control input u(k) (uncertain nonlinear system)



Fig. 7 Evolution of the sliding function S(k) (uncertain nonlinear system)

It can be seen that the inclusion of uncertainties increases the chattering phenomenon.

D. The Neuronal Discrete Sliding Mode Control for inverted pendulum with uncertainties: NDSMC

The simulation results of the neuro discrete nonlinear sliding mode control are shown in figures 8, 9 and 10. Figure 8 presents the evolution of the state $x_1(k)$ and $x_2(k)$ within the NDSMC comparing with the state $x_1(k)$ and $x_2(k)$ within the classical discrete sliding mode control. Figure 10 shows both the evolution of the neuronal sliding function Sn(k) and the classical sliding function S(k).



Fig. 8 The Evolution of nonlinear system $x_1(k)$ and $x_2(k)$ with DSMC and NDSMC



Fig. 10 Evolution of the Neuro discrete sliding function and Discrete sliding function

From these figures, it is clear that the oscillations encountered in the case of the classical sliding mode control when we introduce an important parametric uncertainty are eliminated. Therefore, the proposed neuro discrete nonlinear sliding mode control law is able to eliminate the chattering phenomenon in spite the presence of uncertainties.

V. CONCLUSIONS

In this work, a new neuro discrete-time sliding mode control for a non-linear system with uncertainties is developed. The application of this control law on an inverted pendulum has given satisfactory results for the stabilization, the trajectory tracking and overcoming the chattering phenomenon comparing with classical sliding mode control.

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REFERENCES

- Z-P. Wang, H-N. Wu, "H∞ sampled-data fuzzy control for non-linear parabolic distributed parameter systems with control inputs missing," *IET Control Theory & Applications.*, 2017, vol. 11, no. 10, pp. 1530-1541.
- [2] R. Zhang, D. Chen., W. Yao, D. Ba, X. Ma, "Non-linear fuzzy predictive control of hydroelectric system," *IET Generation*, *Transmission & Distribution*, 2017, vol. 11, no. 8, pp. 1966-1975.
- [3] G-X. Wen, C-L-P. Chen., Y-J. Liu, Z. Liu, "Neural-network-based adaptive leader-following consensus control for second-order nonlinear multi-agent systems" *IET Control Theory & Applications.*, 2015, vol. 9, no. 13, pp. 1927–1934.
- [4] M. Sakly, A. Sakly and F. M'Shali, "Inverse Optimal Control of Switched Discrete Nonlinear Systems based on Control Lyapunov Function and Genetic Algorithm," in 16th international conference on Sciences and Techniques of Automatic control & computer engineering - STA'2015.
- [5] S. Chaouch, L. Abdou, and L-C Alaoui, "Nonlinear Backstepping Control Using Genetic Algorithm of Induction Motor Without Speed Encoder," in 15th international conference on Sciences and Techniques of Automatic control & computer engineering - STA'2014.
- [6] U. Itkis, "Control systems of variable structure". *John Wiley and Sons*, 1976, New York.
- [7] R. Decarlo, H. Zak and G. Mattews, "Variable structure control of nonlinear multivariable systems: A tutorial", *Proc. IEEE.*, 1988, vol. 73, pp. 212–232.
- [8] V. Utkin, "Variable structure systems with sliding mode", IEEE Transactions on Automatic Control, 1977, vol. 22, no.12, pp. 212-222.
- [9] P. Lopez et A. Nouri, "Théorie élémentaire et pratique de la commande par les régimes glissant Mathématiques et applications ", 2006, SMAI : Springer-Verlag.
- [10] V. Utkin, Sliding Mode in Control and Optimization, Berlin: Springer-Verlag, 1992.
- [11] K. Dehri, M. Ltaief, and A.S. Nouri, "Discrete Second Order Sliding Mode Control for Nonlinear Multivariable Systems," *Electrotechnical Conference (MELECON'12)*, 2012, pp. 387–390.
- [12] A. Znidi, K. Dehri, and A.S. Nouri, "Adaptive Sliding Mode Control for Discrete Uncertain Systems Using Matrix RLS Algorithm," 15th International Conference Sciences and Techniques of Automatic Control and Computer Engineering on IEEE, 2014, pp. 953–957.
- [13] Z. Hajji, K. Dehri, and A.S. Nouri, "Stability Analysis of Discrete Integral Sliding Mode Control for Input–Output Model," Journal of Dynamic Systems, Measurement, and Control, 2017, vol. 139, pp. 1-7.
- [14] H. Romdhane, K. Dehri, and A.S. Nouri, "Second Order Sliding Mode Control for Discrete Decouplable Multivariable Systems via Inputoutput Models," International Journal of Automation and Computing, 2015, vol. 12, no. 6, pp. 630-638.
- [15] N. Essounbouli, A. Hamzaoui and J. Zaytoon, "Fuzzy sliding mode control for a class of non-linear continuous systems" *Int. J. Computer Applications in Technology*, 2006, vol. 27, no. 2/3, pp. 174–182.

- [16] Y. Jiang and B-Y. Jiang, "Multi-model neural network sliding mode control for Robotic Manipulators" in the International Conference on Mechatronics and Control (ICMC), 2014, p. 2431- 2435.
- [17] M. Ahmed, M. A. Ebrahim, H. S. Ramadan and M. Becherif, "Optimal genetic-sliding mode control of VSC-HVDC transmission systems" in International Conference on Technologies and Materials for Renewable Energy, Environment and Sustainability, 2015, pp. 1048 -1060.
- [18] F. Qiao, Q. Zhu, A. Winfield et C. Melhuish, "Fuzzy sliding mode control for discrete nonlinear systems," Transactions of China Automation Society, 2003, vol. 22, no.12, pp. 311-315.
- [19] X. Yu and Z. Man." Variable structure systems with terminal sliding modes", Berlin, Springer, 2002, pp. 109-127.
- Y. Wang W. Gao and A. Homaifa." Discrete-time variable structure [20] control systems". IEEE Transaction on Industrial Electronics, vol. 42, no. 2, 1995, pp. 117-122.
- [21] H. Morioka, K. Wada, A. Sabanovic, "Neural network based chattering free sliding mode control". Proceeding of the 34th SICE anniversary conference, 1995, pp. 1303-1308.
- [22] S. Tzafestas, "Neural Networks in Robotics: State of the Art," IEEE Int.Conf On Industrial Electronics, 1995.
- [23] H. Hu, P. Y. Woo. "Fuzzy supervisory sliding mode and neural network control for manipulators ", IEEE Tans on industry and
- *electronics*, 2006, vol.53, no.3, pp. 929-940. X. Mu and Q. Li, "High resolution fiber distributed measurements with coherent OFDR," in *the 8th World Congress on Intelligent Control and* [24] Automation, 2010, Jinan, China, pp. 6610.
- R. Rojas, Neural Networks, Springer-Verlag, Berlin, 1996 [25]
- [26] R. Ben Mohamed, H. Ben Nasr and F. M'Sahli, "A multimodel approach for a nonlinear system based on neural network validity," International Journal of Intelligent Computing and Cybernetics, 2011, vol. 4, no.3, pp. 331-352.