

State and Unknown Inputs Estimation for a Class of Discrete-time Takagi-Sugeno Descriptor Models

Karim Bouassem^{1,2}, Jalal Souлами^{1,2}, Abdellatif El Assoudi^{1,2}, El Hassane El Yaagoubi^{1,2}

¹Laboratory of High Energy Physics and Condensed Matter, Faculty of Science
 Hassan II University of Casablanca, B.P 5366, Maarif, Casablanca, Morocco

²ECPI, Department of Electrical Engineering, ENSEM
 Hassan II University of Casablanca, B.P 8118, Oasis, Casablanca Morocco.
 Email: jalal.soulami@gmail.com

Abstract—In this paper, the design problem of simultaneous estimation of unmeasurable states and unknown inputs (UIs) is investigated for a class of discrete-time Takagi-Sugeno descriptor models (DTSDMs) with measurable premise variables. The UIs affect both state and output of the system. The approach is based on the separation between dynamic and static relations in the considered DTSDM. First, the method permitting to separate dynamic equations from static equations is exposed. Next, an augmented fuzzy explicit model which contains the dynamic equations and the UIs is constructed. Then a fuzzy unknown inputs observer (FUIO) design in explicit structure is developed. The exponential convergence of the state estimation error is studied by using the Lyapunov theory and the stability conditions are given in terms of linear matrix inequalities (LMIs). Finally, an illustrative example is given to show the good performances of the proposed method.

Keywords: Discrete-time Takagi-Sugeno descriptor model, unknown inputs, fuzzy unknown inputs observer, LMI.

In this paper, some notations used are fair standard.

For example, $X > 0$ means the matrix X is symmetric and positive definite. X^T denotes the transpose of X .

The symbol I (or 0) represents the identity matrix (or zero matrix) with appropriate dimension.

$$\sum_{i,j=1}^q \mu_i \mu_j = \sum_{i=1}^q \sum_{j=1}^q \mu_i \mu_j, \begin{pmatrix} X & * \\ Z & Y \end{pmatrix} = \begin{pmatrix} X & Z^T \\ Z & Y \end{pmatrix}.$$

I. INTRODUCTION AND PROBLEM STATEMENT

Descriptor dynamic models, known as a generalization of standard dynamic models, constitute a powerful modeling tool allowing to describe the dynamic behavior of processes governed by both dynamic and static equations. They represent physical phenomenas that can not be described by standard models, see [1], [2], [3] for some real applications of descriptor models. Moreover, the ordinary T-S fuzzy model [4], [5] has been successfully developed to study nonlinear control systems, see e.g. [6], [7] and the references therein. In [8], [9], a fuzzy descriptor system is defined by extending the T-S fuzzy model [4]. Notice that, UIs can result either from uncertainty in the model or from the presence of unknown external excitation. Thus, due to the increasing demand for reliability and maintainability of the automatic control process,

unknown inputs observer design is widely used in the area of fault detection and design of fault tolerant control strategy. This is one of the most attractive research areas in both theoretical and practical fields during these last two decades, see e.g. [10], [11], [12] for works using different approaches. In this paper, the following class of DTSDMs subject to UIs which affect both state and output of the system is considered:

$$\begin{cases} MZ_{k+1} &= \sum_{i=1}^q \mu_i(\eta_k)(A_i Z_k + B_i u_k + C_i d_k) \\ y_k &= \sum_{i=1}^q \mu_i(\eta_k)(D_i Z_k + E_i u_k + F_i d_k) \end{cases} \quad (1)$$

where $Z_k^T = [Z_k^1 \quad Z_k^2]^T \in \mathbf{R}^n$ is the state vector with $Z_k^1 \in \mathbf{R}^{n_1}$ is the vector of difference variables, $Z_k^2 \in \mathbf{R}^{n_2}$ is the vector of algebraic variables with $n_1 + n_2 = n$, $u_k \in \mathbf{R}^m$ is the control input, $d_k \in \mathbf{R}^r$ is the unknown control input, $y_k \in \mathbf{R}^p$ is the measured output. $A_i \in \mathbf{R}^{n \times n}$, $B_i \in \mathbf{R}^{n \times m}$, $C_i \in \mathbf{R}^{n \times r}$, $D_i \in \mathbf{R}^{p \times n}$, $E_i \in \mathbf{R}^{p \times m}$, $F_i \in \mathbf{R}^{p \times r}$, $M \in \mathbf{R}^{n \times n}$ such that $\text{rank}(M) = n_1$ are real known constant matrices with:

$$M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}; \quad A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix} \quad (2)$$

$$B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix}; \quad C_i = \begin{pmatrix} C_{1i} \\ C_{2i} \end{pmatrix}; \quad D_i = \begin{pmatrix} D_{1i} & D_{2i} \end{pmatrix} \quad (3)$$

where constant matrices A_{22i} are supposed invertible. q is the number of sub-models. η_k is the premise variable which is supposed here to be real-time accessible and the $\mu_i(\eta_k)$ ($i = 1, \dots, q$) are the weighting functions that ensure the transition between the contribution of each sub model:

$$\begin{cases} MZ_{k+1} &= A_i Z_k + B_i u_k + C_i d_k \\ y_k &= D_i Z_k + E_i u_k + F_i d_k \end{cases} \quad (4)$$

They verify the so-called convex sum properties:

$$\begin{cases} \sum_{i=1}^q \mu_i(\eta_k) = 1 \\ 0 \leq \mu_i(\eta_k) \leq 1 \quad i = 1, \dots, q \end{cases} \quad (5)$$

The aim of the paper consists in investigating the problem of FUIO design for the class of systems (1). Notice that, this problem design for T-S explicit or descriptor systems has received considerable attention and is still an active area of research in both continuous-time and discrete-time cases. Indeed, for T-S fuzzy systems described by ordinary dynamic equations subject to UIs, various developments on fuzzy observer and its application to fault detection exist in the literature, see for instance [13], [14], [15], [16], [17] for continuous-time systems and [18], [19], [20], [21] for discrete-time systems. Likewise, for T-S fuzzy descriptor systems subject to UIs several works are discussed in the literature see e.g. [22], [23], [24], [25], [26], [27], [28]. It should be noted that, generally, an interesting way to solve the various FUIO raised previously is to write the convergence conditions on the LMI form [29].

Before giving the main result, let us make the following assumption [1], [24]:

Assumption 1: : Suppose that:

- (M, A_i) is regular, i.e. $\det(zM - A_i) \neq 0 \forall z \in \mathbf{C}$
- All sub-models (4) are impulse observable and detectable.

In order to investigate the FUIO design for system (1), we proceed as mentioned above to the separation of the dynamic equations from static equations of the model (1). Indeed, from (2)-(3), sub-model (4) can be rewritten as follows:

$$\begin{cases} Z_{k+1}^1 &= A_{11i}Z_k^1 + A_{12i}Z_k^2 + B_{1i}u_k + C_{1i}d_k \\ 0 &= A_{21i}Z_k^1 + A_{22i}Z_k^2 + B_{2i}u_k + C_{2i}d_k \\ y_k &= D_{1i}Z_k^1 + D_{2i}Z_k^2 + E_i u_k + F_i d_k \end{cases} \quad (6)$$

Since A_{22i} is invertible, it follows:

$$Z_k^2 = J_i Z_k^1 + K_i u_k + L_i d_k \quad (7)$$

where

$$\begin{cases} J_i &= -A_{22i}^{-1}A_{21i} \\ K_i &= -A_{22i}^{-1}B_{2i} \\ L_i &= -A_{22i}^{-1}C_{2i} \end{cases} \quad (8)$$

Thus, combining (6) and (7) we have:

$$\begin{cases} Z_{k+1}^1 &= M_i Z_k^1 + N_i u_k + P_i d_k \\ Z_k^2 &= J_i Z_k^1 + K_i u_k + L_i d_k \\ y_k &= R_i Z_k^1 + S_i u_k + T_i d_k \end{cases} \quad (9)$$

where

$$\begin{cases} M_i &= A_{11i} + A_{12i}J_i \\ N_i &= B_{1i} + A_{12i}K_i \\ P_i &= C_{1i} + A_{12i}L_i \\ R_i &= D_{1i} + D_{2i}J_i \\ S_i &= E_i + D_{2i}K_i \\ T_i &= F_i + D_{2i}L_i \end{cases} \quad (10)$$

So, by aggregation of the resulting sub-models (9), the following global fuzzy model is obtained:

$$\begin{cases} Z_{k+1}^1 &= \sum_{i=1}^q \mu_i(\eta_k)(M_i Z_k^1 + N_i u_k + P_i d_k) \\ Z_k^2 &= \sum_{i=1}^q \mu_i(\eta_k)(J_i Z_k^1 + K_i u_k + L_i d_k) \\ y_k &= \sum_{i=1}^q \mu_i(\eta_k)(R_i Z_k^1 + S_i u_k + T_i d_k) \end{cases} \quad (11)$$

Assumption 2: : Suppose that d_k is considered as a constant unknown control input per time interval i.e.:

$$d_{k+1} = d_k \quad k \in [T_1 \ T_2], \quad \forall T_1, T_2 \in \mathbf{R}^+ \quad (12)$$

Let us define the augmented state vector $\xi_k^1 = [Z_k^1 \ d_k^T]^T$ and $\xi_k^2 = Z_k^2$. Thus, the system (11) can be represented as:

$$\begin{cases} \xi_{k+1}^1 &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k) \\ \xi_k^2 &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{J}_i \xi_k^1 + K_i u_k) \\ y_k &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{R}_i \xi_k^1 + S_i u_k) \end{cases} \quad (13)$$

where

$$\begin{cases} \tilde{M}_i &= \begin{pmatrix} M_i & P_i \\ 0 & I \end{pmatrix} \\ \tilde{N}_i &= \begin{pmatrix} N_i \\ 0 \end{pmatrix} \\ \tilde{J}_i &= \begin{pmatrix} J_i & L_i \end{pmatrix} \\ \tilde{R}_i &= \begin{pmatrix} R_i & T_i \end{pmatrix} \end{cases} \quad (14)$$

The rest of the paper is structured as follows. The main result about FUIO design permitting to estimate simultaneously unmeasurable states and UIs for the considered class of systems (1) is stated in Section 2. The observer gains are found directly from LMI formulation. In Section 3, a numerical example to show the good performance of the proposed technique is given. Finally, a conclusion is given in section 4.

II. STATE AND UNKNOWN INPUTS ESTIMATION

Systems (1) and (13) are equivalent. For the design of the fuzzy observer permitting to estimate simultaneously the unmeasurable states and UIs, we will use the second structure. So, the proposed FUIO takes the following form:

$$\begin{cases} \hat{\xi}_{k+1}^1 &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{M}_i \hat{\xi}_k^1 + \tilde{N}_i u_k - G_i(\hat{y}_k - y_k)) \\ \hat{\xi}_k^2 &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k) \\ \hat{y}_k &= \sum_{i=1}^q \mu_i(\eta_k)(\tilde{R}_i \hat{\xi}_k^1 + S_i u_k) \end{cases} \quad (15)$$

where $(\hat{\xi}_k^1, \hat{\xi}_k^2)$ and \hat{y}_k denote the estimated augmented state vector and the output vector respectively. The activation

functions $\mu_i(\eta_k)$ are the same than those used in the T-S model (13). G_i , $i = 1, \dots, q$ are the gains of FUIO which are determined such that $(\hat{\xi}_k^1, \hat{\xi}_k^2)$ asymptotically converges to (ξ_k^1, ξ_k^2) .

In order to establish the conditions for the asymptotic convergence of the observer (15), we define the state estimation error:

$$\varepsilon_k = \begin{pmatrix} \varepsilon_k^1 \\ \varepsilon_k^2 \end{pmatrix} = \begin{pmatrix} \hat{\xi}_k^1 - \xi_k^1 \\ \hat{\xi}_k^2 - \xi_k^2 \end{pmatrix} \quad (16)$$

It follows from (13) and (15) that the estimated error equation can be written as:

$$\begin{cases} \varepsilon_{k+1}^1 = \sum_{i,j=1}^q \mu_i(\eta_k)\mu_j(\eta_k)\Omega_{ij}\varepsilon_k^1 \\ \varepsilon_k^2 = \sum_{i=1}^q \mu_i(\eta_k)Q_i\varepsilon_k^1 \end{cases} \quad (17)$$

where

$$\Omega_{ij} = \tilde{M}_i - G_i\tilde{R}_j \quad (18)$$

To prove the convergence of the estimation error ε_k toward zero, it suffices to prove from (17), that ε_k^1 converges toward zero. The main result is stated in the following Theorem.

Theorem 1: : There exists an FUIO (15) for DTSDM (1) if given $0 < \alpha < 1$ there exist matrices $Q > 0$, W_i , $i = 1, \dots, q$ verifying the following LMIs:

$$\begin{pmatrix} -\alpha^2 Q & * \\ Q\tilde{M}_i - W_i\tilde{R}_j & -Q \end{pmatrix} < 0 \quad \forall i, j \in \{1, \dots, q\} \quad (19)$$

The fuzzy local observer gains G_i , $i = 1, \dots, q$ are given by:

$$G_i = Q^{-1}W_i \quad (20)$$

Proof of Theorem 1 : Let us consider the following quadratic Lyapunov function as follows:

$$V_k = (\varepsilon_k^1)^T Q \varepsilon_k^1, \quad Q > 0 \quad (21)$$

Estimation error convergence is exponentially ensured if the following condition is guaranteed ([30] as cited in [6]):

$$\begin{aligned} V_{k+1} - V_k &= (\varepsilon_{k+1}^1)^T Q \varepsilon_{k+1}^1 - (\varepsilon_k^1)^T Q \varepsilon_k^1 \\ &< (\alpha^2 - 1)V_k \end{aligned} \quad (22)$$

with $0 < \alpha < 1$.

By using (17), the condition (22) can be written as:

$$\begin{aligned} V_{k+1} - V_k &= \sum_{i,j=1}^q \mu_i(\eta)\mu_j(\eta)(\varepsilon_k^1)^T (\Omega_{ij}^T Q \Omega_{ij} - Q) \varepsilon_k^1 \\ &< (\alpha^2 - 1)V_k \end{aligned} \quad (23)$$

which is equivalent to the following stability conditions:

$$\Omega_{ij}^T Q \Omega_{ij} - \alpha^2 Q < 0 \quad i, j = 1, \dots, q \quad (24)$$

Letting $W_i = QG_i$, from (18) it follows that (24) is equivalent to (19) by using the Schur complement [29]. From the Lyapunov stability theory, if the LMI conditions (19) are satisfied, the error dynamic equation (17) is exponentially asymptotically stable.

III. NUMERICAL ILLUSTRATION

In order to show the performance of the proposed method of FUIO design, the following DTSDM is considered:

$$\begin{cases} MZ_{k+1} = \sum_{i=1}^2 \mu_i(\eta_k)(A_i Z_k + B u_k + C d_k) \\ y = DZ \end{cases} \quad (25)$$

where $Z_k = (z_{1k}, z_{2k}, z_{3k}, z_{4k})^T \in \mathbf{R}^4$, $u_k \in \mathbf{R}$, $d_k \in \mathbf{R}$ and $y_k \in \mathbf{R}$ are the state vector, known input, UI and output, respectively. The matrices numerical values are:

$$\begin{aligned} A_1 &= \begin{pmatrix} 1 & 0.0100 & 0 & 0 \\ -0.0250 & 0.9925 & 0 & 0.0003 \\ 0 & 0.0100 & -0.0040 & 0 \\ -0.0250 & -0.0075 & 0 & 0.0008 \end{pmatrix} \\ A_2 &= \begin{pmatrix} 1 & 0.0100 & 0 & 0 \\ -0.0270 & 0.9925 & 0 & 0.0003 \\ 0 & 0.0100 & -0.0040 & 0 \\ -0.0270 & -0.0075 & 0 & 0.0008 \end{pmatrix} \\ M &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -0.0013 \end{pmatrix} \\ C &= \begin{pmatrix} 0 \\ 0.01 \\ 0 \\ 0 \end{pmatrix}; \quad D = (1 \ 0 \ 0 \ 0) \end{aligned}$$

The weighting functions are:

$$\mu_1(\eta_k) = 1 - 12.76 * z_{1k}^2 \quad \text{and} \quad \mu_2(\eta_k) = 12.76 * z_{1k}^2.$$

Therefore to apply the proposed FUIO (18) for the model (38), as stated in Theorem 1, it suffices to rewrite the model (25) into its equivalent form (13) as mentioned above.

Thus, by Theorem 1 with $\alpha = 0.92$ the following observer gains G_1 and G_2 are obtained:

$$G_1 = \begin{pmatrix} 1.3621 \\ 29.2238 \\ 190.0753 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 1.3620 \\ 29.2208 \\ 190.0639 \end{pmatrix}$$

The expression of unknown input signal d_k is defined as in Figure 1 and the input signal u_k is defined as:

$$u_k = \begin{cases} 2 & 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

Simulation results with initial conditions:

$$\begin{aligned} \xi_k^1 &= [0.10 \ 0.30 \ 2.00]^T, & \xi_k^2 &= [0.75 \ 3.03]^T \\ \hat{\xi}_k^1 &= [0.10 \ 0.45 \ 4.00]^T, & \hat{\xi}_k^2 &= [1.13 \ 4.53]^T \end{aligned}$$

are given in Figures 1 to 5. These simulation results show the performances of the proposed FUIO (15) with the gains G_1 , G_2 where the dashed lines denote the state variables and UI estimated by the FUIO. They show that the FUIO gives a good estimation of unmeasurable states and UI of the considered DTSDM.

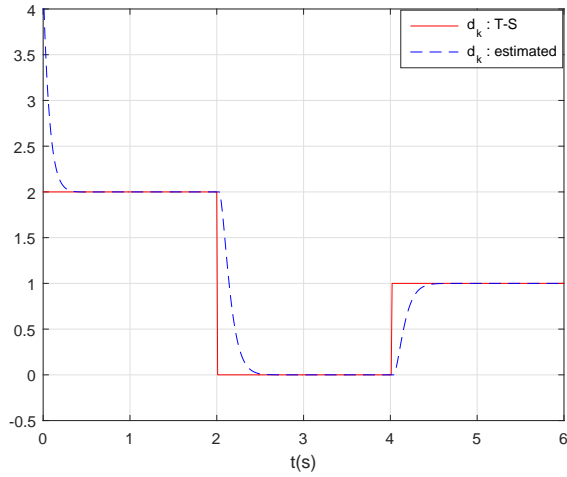


Fig. 1. Unknown input d_k and its estimate

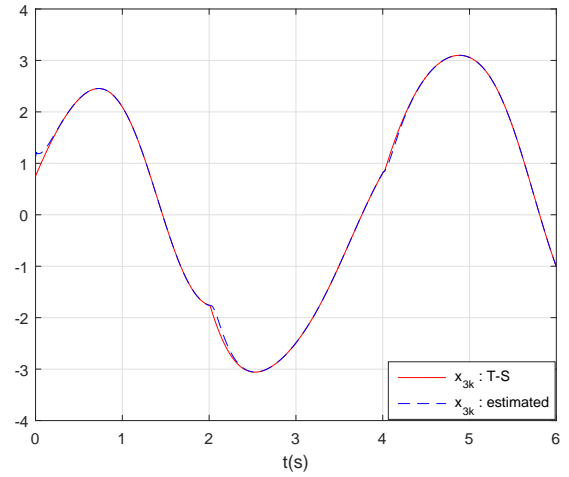


Fig. 4. State variables x_{3k} and its estimate

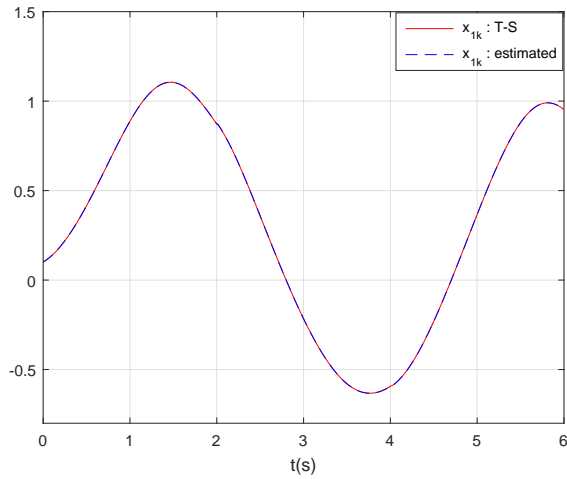


Fig. 2. State variables x_{1k} and its estimate

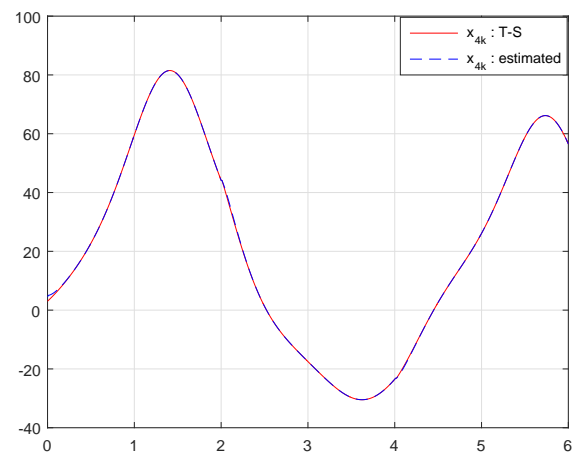


Fig. 5. State variables x_{4k} and its estimate

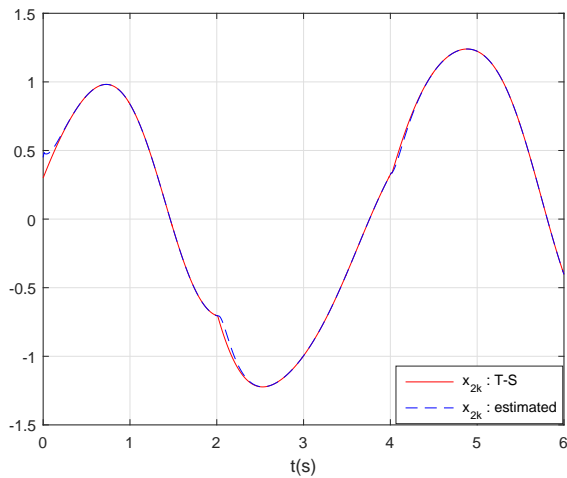


Fig. 3. State variables x_{2k} and its estimate

IV. CONCLUSION

A novel method of fuzzy observer design for a class of DTSIMs with measurable premise variables and subject to UIs which affect both state and output of the model is presented in this paper. The proposed result permitting to estimate simultaneously the system state and the UIs is based on the separation between dynamic and static equations in the considered fuzzy descriptor model and the use of an augmented system structure formed by the dynamic equations and the UIs. The exponential convergence of the state estimation error is studied by using the Lyapunov theory and the existence of the condition ensuring this convergence is expressed in term of LMIs. To show the good performance of the proposed method, a DTSDM subject to UI variable is proposed. The effectiveness of the proposed FUIO design for the on-line simultaneous estimation of unknown states and UI of the considered model is illustrated by numerical simulation, since both state and UI are well estimated.

REFERENCES

- [1] L. Dai. Singular Control Systems. Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, 1989.
- [2] A. Kumar and P. Daoutidis, Control of nonlinear differential algebraic equation systems. Chapman & Hall CRC, 1999.
- [3] G. R. Duan. Analysis and Design of Descriptor Linear Systems. Springer 2010.
- [4] T. Takagi, M. Sugeno, Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst., Man and Cyber, Vol.1115, pp. 116-132, 1985.
- [5] T. Taniguchi, K. Tanaka, H. Ohtake and H. Wang, Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems. IEEE Transactions on Fuzzy Systems, vol. 9, No.4, pp. 525-538, August 2001.
- [6] K. Tanaka, and H. O. Wang, Fuzzy control systems design and analysis: A Linear Matrix Inequality Approach. John Wiley & Sons; 2001.
- [7] Zs. Lendek, T. M. Guerra, R. Babuka and B. De Schutter. Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models, Springer Berlin Heidelberg, 2011.
- [8] T. Taniguchi, K. Tanaka, K. Yamafuji and H. O. Wang, Fuzzy Descriptor Systems: Stability Analysis and Design via LMIs. Proceedings of the American Control Conference. San Diego, California June 1999, pp. 1827-1831..
- [9] T. Taniguchi, K. Tanaka, and H. O. Wang, Fuzzy Descriptor Systems and Nonlinear Model Following Control, IEEE Transactions on Fuzzy Systems, vol. 8, No.4, pp. 442-452, August 2000.
- [10] R. Isermann, Fault-Diagnosis Systems An Introduction from Fault Detection to Fault Tolerance. Springer-Verlag Berlin Heidelberg 2006.
- [11] Z. Lendek, T. M. Guerra, R. Babuka and B. De Schutter, Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models. Springer-Verlag Berlin Heidelberg 2010.
- [12] L. Li, Fault Detection and Fault-Tolerant Control for Nonlinear Systems. Springer Fachmedien Wiesbaden 2016.
- [13] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, State and unknown input estimation for nonlinear systems described by Takagi-Sugeno models with unmeasurable premise variables. 17th Mediterranean Conference on Control and Automation MED'09, June 24-26, 2009, Makedonia Palace, Thessaloniki, Greece.
- [14] D. Ichalal; B. Marx; J. Ragot; D. Maquin, New fault tolerant control strategies for nonlinear Takagi-Sugeno systems. International Journal of Applied Mathematics and Computer Science, 22(1): 197-210, 2012.
- [15] M. Kamel, M. Chadli, M. Chaabane, Unknown inputs observer for a class of nonlinear uncertain systems; An LMI approach. International Journal of Automation and Computing, 9(3), June 2012, 331-336.
- [16] M. Chadli, and H.R. Karimi, Robust observer design for unknown inputs Takagi-Sugeno Models. IEEE Transactions on Fuzzy Systems, 21(1): 158-164, 2013.
- [17] T. Youssef, M. Chadli and M. Zelmat, Synthesis of a unknown inputs proportional integral observer for TS fuzzy models. European Control Conference (ECC), July 17-19, 2013, Zurich, Switzerland.
- [18] M. Chadli, A .Akhenak, J .Ragot, D .Maquin, State and unknown input estimation for discrete-time multiple model. Journal of the Franklin Institute 346, 593-610, 2009.
- [19] E. Maherzi, M. Besbes, S. Zimmel, A. Mami, Estimation of the State and the Unknown Inputs of a Multimodel with non Measurable Decision Variables. Journal of Applied Research and Technology (JART), vol. 12(3), pp. 422-434, June 2014.
- [20] D. Rotondo, M. Witczak, V. Puig, F. Nejjari & M. Pazera, Robust unknown input observer for state and fault estimation in discrete-time Takagi-Sugeno systems, International Journal of Systems Science 2016.
- [21] K. Zhang, B. Jiang, V. Cocquempot, Fuzzy unknown input observer-based robust fault estimation design for discrete-time fuzzy systems. Signal Processing, Volume 128, Pages 40-47, November 2016.
- [22] V. Estrada-Manzo, Z. Lendek, T. M. Guerra, Discrete-time Takagi-Sugeno descriptor models: observer design. IFAC, Cape Town, South Africa. August 24-29, 2014.
- [23] B. Marx, D. Koenig and J. Ragot, Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis. IET Control Theory and Applications, October 2007.
- [24] C. Mechmech, H. Hamdi, M. Rodrigues, and N. Benhadj Braiek, State and unknown inputs estimations for multi-models descriptor systems. American Journal of Computational and Applied Mathematics 2012, 2(3): 86-93.
- [25] F. R. Lopez-Estrada, J.C. Ponsart, Didier Theilliol, C. M.Astorga-Zaragoza, S. Aberkane, Fault Diagnosis Based on Robust Observer for Descriptor-LPV Systems with Unmeasurable Scheduling Functions. 19th IFAC World Congress. Cape Town, South Africa. August 24-29, 2014.
- [26] K. Bouassem, J. Soulami, A. El Assoudi and E. El Yaagoubi, Unknown Input Observer Design for a Class of Takagi-Sugeno Descriptor Systems. Nonlinear Analysis and Differential Equations, Vol. 4, 2016, no. 10, 477-492. <http://dx.doi.org/10.12988/nade.2016.6635>.
- [27] A. Louzimi, A. El Assoudi, J. Soulami and E. El Yaagoubi, Unknown Input Observer Design for a Class of Nonlinear Descriptor Systems: A Takagi-Sugeno Approach with Lipschitz Constraints. Nonlinear Analysis and Differential Equations, Vol. 5, 2017, no. 3, 99-116.
- [28] K. Bouassem, J. Soulami, A. El Assoudi and E. El Yaagoubi, Fuzzy Observer Design for a Class of Takagi-Sugeno Descriptor Systems Subject to Unknown Inputs. Nonlinear Analysis and Differential Equations, Vol. 5, 2017, no. 3, 117-134.
- [29] S. Boyd, E. Ghaoui, E. Feron and V. Balakrishnan, Linear matrix inequalities in system and control theory. Society for Industrial and Applied Mathematics, Philadelphia, 1994.
- [30] A. Ichikawa et al., Control Hand Book, Ohmu Publisher, 1993, Tokyo in Japanese.