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Modeling of Rectangular Waveguide without Infinite Flange

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***Abstract*— To model an open ended waveguide radiating in the free space, we usually consider it as a waveguide opened in an infinite metal flange in order to eliminate the rear radiation. In this work another approach is proposed for modeling it without flange. The basic idea is to use a simple concentric discontinuity in a rectangular waveguide, and determine the adequate dimensions of this waveguide to properly simulate the free space. The proposed electromagnetic analysis is based on moments method combined to the generalized equivalent circuit method (MoM-GEC). The S parameters and the electromagnetic field are investigated and compared to those published in literature, a good agreement is shown.**

Keywords*— Modeling, Waveguide, MoM-GEC, Propagation, Scattering.

I. INTRODUCTION

Open-ended rectangular waveguide is the elementary component for the millimetre wave's propagation. Several kinds of design have been proposed (single, coupled, networked ...) to be used to achieve many applications such as satellite communication, radar, medical systems, geophysical applications [1-3]. The variational method is the first approach used to solve the electromagnetic problem concerning these areas [4-9]. The boundary conditions require a relationship between the tangential electromagnetic fields, which leads to an integral equation [5]. This integral equation is solved using the moment method to investigate the single or coupled waveguides behaviours. Miles [6] considers that the power flows across aperture has a variational form, which is fixed with respect to the first-order variation in the aperture field. Galejs [7] studies the rectangular cavity backed slot and focus on limitations of slot performance in conjunction with small cavities.

There are many other methods which have been applied to solve the flanged waveguide problem. The correlation matrix method which is based on the energy conservation low has been applied in [8], while Teodoridis et al [9] used the characteristic modes and the problem is solved by investigating the eigenvalues. Other methods which based on modal analysis were considered [10]. Boudrant et al. have obtained an integral equation by using the operator transverse method [11]. That integral equation was solved using the Galerkin's method.

In this paper, a new rigorous approach is proposed to model an opened waveguide radiating in the free space. The concept is greatly simple; it consists of using a simple concentric discontinuity in a rectangular waveguide, and determining the adequate dimensions of the waveguide in question to properly simulate the free space. Different cases are discussed and only one is maintained. For validation purpose, the results are compared to those obtained in previous studies. Agood agreement is shown.

II. MODELLING OF STUDIED STRUCTURE

The structure under consideration is formed by a small waveguide, who radiate in another waveguide which aims to simulate the free space. As illustrated in Fig. 1, this structure is looks like a lossless waveguide of infinite length, with a concentric discontinuity. The excitation source is applied in infinity of smaller waveguide in order to generate only fundamental mode f_0. The smallest waveguide is called Real Guide (RG), and the biggest is called Space Guide (SG). The RG is always metallic, in fact is a reel waveguide but the SG is not. So, we propose to discuss the nature of this latter in order to understand his influence.

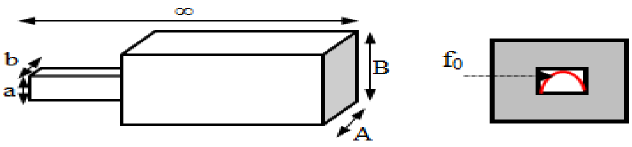

Fig. 1 Modeling Structure

As shown in Fig.2, different cases arise to define the wall's nature composing the SG and the transversal wall which ties SG to RG.

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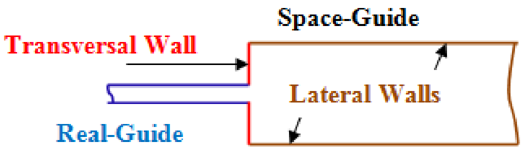

Fig. 2 Frontal view of studied structure.

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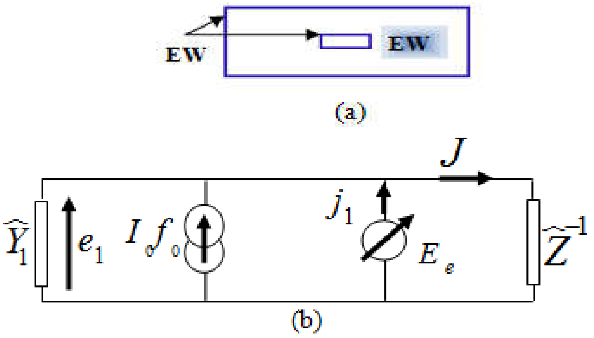
So, RG is presented by electric walls (EW), while SG can be presented by EW or magnetic walls (MW). In totally, there are four possible configurations depending on the Transversal/lateral walls: MW/EW, EW/MW, EW/EW and MW/MW.

The first two cases are omitted because tangential component of E-field has a singularity at the interface EW/MW. Therefore, we have two cases to consider: EW/EW and MW/MW.

III. PROBLEM FORMULATION

A. Space modeling with electric waveguide

As shown in Fig. 3(a), the consider structure is composed by two metallic waveguides tied by transversal metallic wall. In Fig. 3(b), we present the relative equivalent circuit model.



(a)

(b)

Fig. 3 (a) Modeling of space with electric waveguide. (b) The relative equivalent circuit.

The The excitation is brought back to discontinuity plan as a modal source of current, its value is the current density of the fundamental mode $I_0 f_0$, and its internal admittance is $\hat{Y}_1$. This last represents the evanescent modes contribution of RG; and its formal writing is:

$$\hat{Y}_1 = \sum \left| f_{pq} \right\rangle y_{pq} \left\langle f_{pq} \right| \tag{1}$$

.

Where f_{pq} is the RG modal basis without the fundamental f_0, and y_{pq} is the mode admittance of each f_{pq} [12]. The voltage at terminals of this source is e_1, who is its dual greatness.

E_e is the virtual voltage source defined in discontinuity plan, and j_1 is the current flowing it. E_e is the unknown problem, and it is expressed as a serial of test functions g_p weighted by unknown modal amplitudes x_p [13]:

$$E_e = \sum_p x_p g_p \tag{2}$$

The modes contribution of SG is expressed in the discontinuity plane by admittance operator $\hat{Z}^{-1}$.

$$\hat{Z}^{-1} = \sum \left| F_{mn} \right\rangle y_{mn} \left\langle F_{mn} \right| \tag{3}$$

Where F_{mn} is the SG modal basis and y_{mn} is the relative mode admittance.

Based on the equivalent circuit shown in Fig. 3, the integral equations associated with the problem can be easily derived by applying generalized Kirchhoff laws:

$$\begin{cases} J = \widehat{Z}^{-1} E_e \\ J = -\widehat{Y}_1 E_e + I_0 f_0 + j_1 \end{cases} \tag{4}$$

Using the equations system (4), we can write:

$$\widehat{Z}^{-1} E_e = -\widehat{Y}_1 E_e + I_0 f_0 + j_1 \tag{5}$$

Then, we can deduce the current j_1:

$$j_1 = (\widehat{Y}_1 + \widehat{Z}^{-1}) E_e - I_0 f_0 \tag{6}$$

Taking into account the expression $e_1 = E_e$, we can find the relation between source variables and their duals:

$$\begin{pmatrix} e_1 \\ j_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \widehat{Z}^{-1} + \widehat{Y}_1 & -1 \end{pmatrix} \begin{pmatrix} E_e \\ I_0 f_0 \end{pmatrix} \tag{7}$$

Applying the Galerkin's method:

$$\begin{cases} V_0 = \langle f_0 | E_e \rangle = \sum_q x_q \langle f_0 | g_q \rangle = A^t X \\ -I_0 \langle g_p | f_0 \rangle + \sum_q \left\langle g_p \middle| (\widehat{Y}_1 + \widehat{Z}^{-1}) g_q \right\rangle x_q = 0 \end{cases} \tag{8}$$

This equations system can be rewritten as:

$$\begin{cases} A^t X = V_0 \\ -A I_0 + [y] X = 0 \end{cases} \tag{9}$$

Where A is the excitation vector, and $[y]$ is the admittance matrix:

$$A = \langle f_0 | g_p \rangle \tag{10}$$

$$[y] = \begin{bmatrix} \left\langle g_{10} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{10} \right\rangle & \cdots & \left\langle g_{10} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{pq} \right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle g_{pq} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{10} \right\rangle & \cdots & \left\langle g_{pq} \middle| (\hat{Y}_1 + \hat{Z}^{-1}) g_{pq} \right\rangle \end{bmatrix} \tag{11}$$

Finally, we deduce the unknown problem:

$$X = [y]^{-1} A I_0 \tag{12}$$

Taking into account relations (8) and (13), we deduce the impedance matrix Z:

$$Z = A^t [y]^{-1} A \tag{13}$$

Therefore, the scattering matrix $[S]$ is expressed using the reduced impedance z and the identity matrix:

$$[S] = (z - I)(z + I)^{-1} \tag{14}$$

B. *Space modeling with magnetic waveguide*

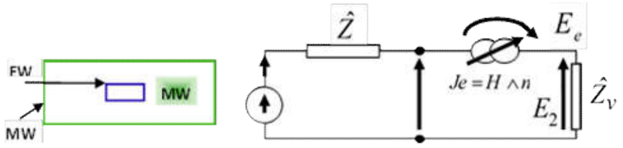


Fig. 4 Modeling of space with magnetic waveguide; and the relative equivalent circuit.

Here, the excitation is presented in discontinuity plan by a voltage modal source, where the transverse electric field of the fundamental mode E_0 is its value, and $\hat{Z}$ is its internal Impedance. $\hat{Z}$ represents the evanescent modes contribution of RG, and has the follow formal writing:

$$\hat{Z}=\sum \left| f_{pq} \right\rangle z_{pq} \left\langle f_{pq} \right| \tag{15}$$

And in the same way as case 1, we determine the internal admittance matrix:

$$Y = A^{t}\left[\hat{Z}+\hat{Z}_{v}\right]^{-1} A \tag{16}$$

And deduce the S parameters.

IV. NUMERICAL RESULTS

In this section, we propose a quantitative discussion about the radiation of a real opened waveguide (WR90) in the free space. In order to determine the appropriate model that simulates the free space, two cases are shown and discussed: Modeling of space with electric Fig. 3, and magnetic waveguide Fig. 4.

To set some parameters of the problem, a convergence study is strongly required. Fig. 5 present the S11 norm as a function of a mode number for different test function number. As shown in Fig. 5(a), the convergence in the first configuration is obtained for 15 test functions and 1600 basis functions (Modal basis of operators). However, in second configuration the convergence is obtained for 20 test functions and 2200 basis functions Fig. 5(b). To ensure the convergence, in the following, we always use these last values for each model

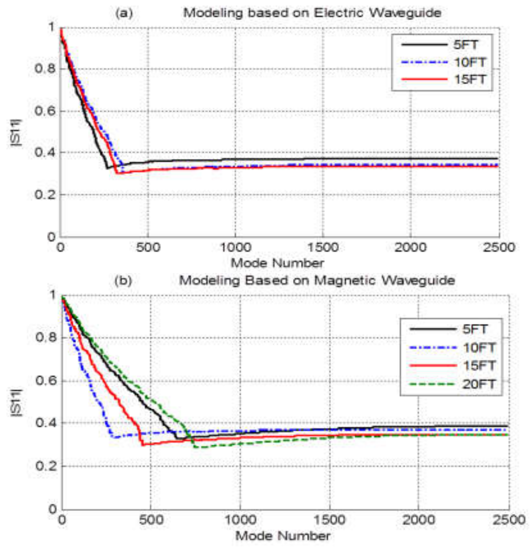


Fig. 5 S11 parameter as a function of mode number. (a) Modeling based on electric waveguide, (b) modeling based on magnetic waveguide.

We consider D and d respectively as dimensions of SG and NG. The main target of this study is to determine the appropriate SG dimensions, to simulate the free space. For this, we present in Fig. 6 the S parameters in two cases, as function of these dimensions. Based on Fig. 6(a), it can be noted that D does not affect results in the electric model, when the dimensions ratio $r = D/d$ exceeds 7. However, as shown in Fig. 6(b), the magnetic model needs $r = 18$ to obtain the same behavior.

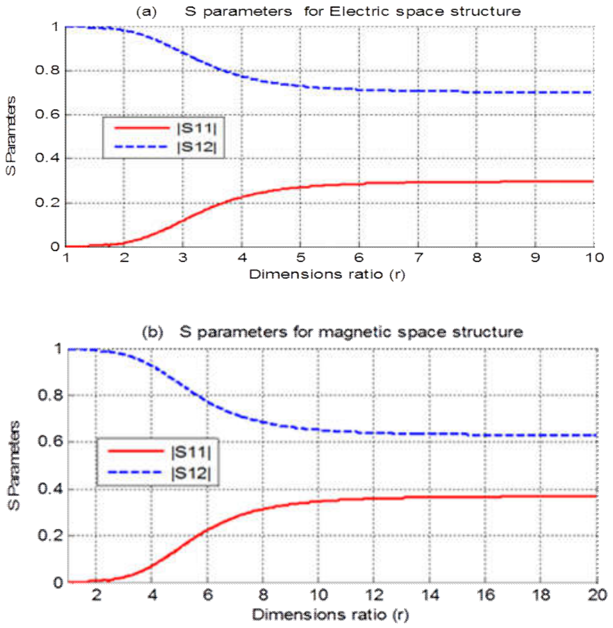


Fig. 6 S parameters based on Space Guide dimensions for: (a) electric space structure, (b) magnetic space structure..

In following, we consider that from D = 7d ~ 4.6 λ there is no more influence of SG on results. For a validation propose, we present in Table 1 the obtained reflection coefficient S_{11} and that obtained in previous works for operating frequency $f = 9.33GHz$. Good agreement is found, and the relative error E_r is lower than10^{-1}.

TABLE I
COMPARISON OF REFLECTION COEFFICIENT WITH PUBLISHED RESULTS

Method	**Re(S11)**	**Im(S11)**	**|S11|**	**E_r**
Present method	0.0768	-0.2662	0.2771	
Baudrand [11]	0.0655	-0.2709	0.2787	0.3%
Mongiardo [14]	0.0803	-0.2365	0.2498	10%

After setting the problem parameters, we propose to study the scattering parameters and electric field distribution in the discontinuity plan.

Fig. 7 shows S_{11} and S_{12} as function of frequency. The SG dimension is fixed to 4.6λ leading a dimension ratio $r = 7$ in order to ensure that SG simulates properly the free space.

As illustrated, S parameters show a resonance frequency $f = 9.33GHz$, which is in agreement with the previous works choice. In this frequency there is a 92.24% of energy transmitted in free space and no mismatch problem is noticed.

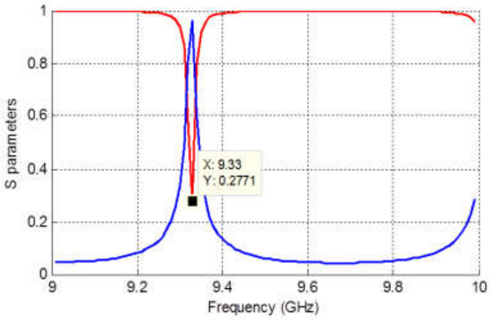


Fig. 7 S parameters based on Frequency.

We present in Fig. 8 the electric field in the discontinuity plan. It shows that, the electric field is concentrated in the RG aperture and vanishes on metal part, which verifies the boundary conditions.

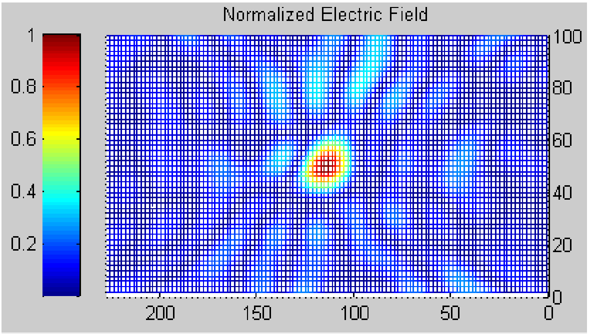


Fig. 8 Normalized electric Field distribution on discontinuity plan.

V. CONCLUSIONS

In this paper, a rigorous formulation based on MoM-GEC method was presented. It uses symmetric bounded operators in order to study the radiation of an open ended waveguide in the free space. Our basic idea consists to simulate the free space as a rectangular waveguide and we seek the appropriate dimensions that do not affect the scattering parameters. Depending on the boundary conditions, four possible structure configurations are proposed and discussed. For a validation purpose, obtained results are compared to previous published works and a good agreement is shown. From the numerical results discussion, it can also be noted that, space modeling based on waveguide with electric walls is the more appropriate configuration that simulates the free space. It requires only a dimensions ratio r=7 and no mismatch problem is shown.

REFERENCES

[1] Hirokawa, Jiro. "Plate-laminated Waveguide Slot Array Antennas and its Polarization Conversion Layers." Automatika–Journal for Control, Measurement, Electronics, Computing and Communications 53.1 (2012).

[2] Douvalis, Vassilis, Yang Hao, and Clive G. Parini. "A monolithic active conical horn antenna array for millimeter and submillimeter wave applications." Antennas and Propagation, IEEE Transactions on 54.5 (2006): 1393-1398.

[3] Lo, Yuen T., and S. W. Lee. Antenna Handbook: theory, applications, and design. Springer Science & Business Media, 2013.

[4] V. Mazauric, "Une approche variationnelle de l'électromagnétisme", 2011.

[5] C. T. Tai, "Application of a variational principle to biconical antennas", *Journal of Applied Physics* 20.11: 1076-1084, 1949.

[6] J. W. Miles, "On the diffraction of an electromagnetic wave through a plane screen", *Journal of Applied Physics,* 20.8: 760-771, 1949

[7] J. Galejs, "Admittance of a rectangular slot which is backed by a rectangular cavity", *IEEE Transactions on Antennas and Propagation*, vol. 11, no 2, pp. 119-126, 1963.

[8] R. H. MacPhie and A. I. Zaghloul, "Radiation from a rectangular waveguide with infinite flange: Exact solution by the correlation matrix method", *IEEE Trans. Antennas Propag.*, vol. AP-28, no. 4, pp. 497–503, Jul. 1980.

[9] V. Teodoridis, T. Sphicopoulos, F. E. Gardiol, "The reflection from an open-ended rectangular waveguide terminated by a layered dielectric medium", *IEEE Trans. Microw. Theory Tech.*, vol. MTT-33, no. 5, pp. 359–366, May 1985.

[10] J. A. Encinar and J. M. Rebollar, "Convergence of numerical solutions of open-ended waveguide by modal analysis and hybrid modal-spectral techniques," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-34, no. 7, pp. 809–814, Jul. 1986.

[11] H. Baudrand, J.-W. Tao, and J. Atechian, "Study of radiating properties of open-ended rectangular waveguides", *IEEE Trans. Antennas Propag.,* vol. 36, no. 8, pp. 1071–1077, Aug. 1988.

[12] H. Baudrand and D. Bajon, "Equivalent circuit representation for integral formulations of electromagnetic problems". *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 15, no 1, pp. 23-57, 2002.

[13] R. F. Harrington and J. L. Harrington, "Field computation by moment methods". *Oxford University Press*, 1996.

[14] M. Mongiardo, T. Rozzi, "Singular integral equation analysis of flange-mounted rectangular waveguide radiators". *IEEE Transactions on Antennas and Propagation*, vol. 41, no 5, pp. 556-565, 1993.